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SÉRIE DE TEXTOS PARA DISCUSSÃO

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Leading by Example: A Simple Evolutionary Approach

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Leadership is usually modeled in a context of international public goods, and the leader or political entrepreneur is an agent whose efforts are directed towards overcoming the free rider problem associated with collective decisions. This entrepreneur can be motivated by the possibility of capturing revenues or votes, or by altruism. A new development in this area is to characterize leadership as a process of leading by example. This means that the leader can generate a Pareto improvement on the non-cooperative outcomes by committing to a minimal level of contribution to the provision of the public good. A key aspect of this kind of analysis is that different results can be obtained depending on the contribution aggregator used. Examples of commonly used aggregators are the summation, weakest-link and best shot aggregators. The literature on the topic of leading by example is incipient and relatively small, and some of its interesting results are based on simple models of non-cooperative simultaneous-move and evolutionary games. The main purpose of this paper is to improve upon those results, investigating more general settings and conditions under which leading by example can support efficient outcomes. In order to do that, it departs from the commonly used numerical approach and derives the Nash equilibria and evolutionary stable strategies of symmetric games where the summation aggregator is used to aggregate individual contributions. It is shown that the possibility of attaining efficient outcomes using the concept of evolutionary stability depends upon conditions pertaining to the concavity of the utility function of the consumption of the public good.

JEL Codes: C72, D71, H41.

I Introduction

The question of how cooperative outcomes can be achieved by means of noncooperative behavior is not new. In particular, the prisoner's dilemma has spawned answers to this question in many directions, including the provision of public goods. In such a context of collective decisions, one easy answer is that, in the real world, some agents act as leaders and have the will and means to bring people together in order to achieve cooperative outcomes. But then, of course, there is the question of how these agents become leaders and how effective their leadership can be.

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Leadership is usually modeled in a context of international public goods[1], and the leader or political entrepreneur is an agent whose efforts are directed towards overcoming the free rider problem associated with collective decisions. This entrepreneur can be motivated, for instance, by the possibility of capturing revenues or votes, or by altruism.

A new development in this area is due to Arce [2001], who characterizes leadership as a process of leading by example[2]. This means that the leader can generate a Pareto improvement on the non-cooperative outcomes by committing to a minimal level of contribution to the provision of the public good and to match higher contributions. A key aspect of this kind of analysis is that different results can be obtained depending on the contribution aggregator used. Examples of commonly used aggregators are the summation, weakest-link and best shot aggregators.

The literature on the topic of leading by example is incipient and relatively small, and some of its interesting results are based on simple models of non-cooperative simultaneous-move and evolutionary games. The main purpose of this paper is to improve upon those results, investigating more general settings and conditions under which leading by example can support efficient outcomes. In order to do that, it departs from the numerical approach in Arce [2001] and derives the Nash equilibria and evolutionarily stable strategies of symmetric games where the summation contribution aggregator is used to aggregate individual contributions. We do not deal here with other types of contribution aggregators, such as weakest-link, weaker-link, best-shot and better-shot, although the methodology developed here could certainly be used to analyze such aggregators.

Right after this introduction, section II briefly reviews some key concepts from the theory of evolutionary stability. Section III presents the main types of contribution aggregators found in the literature, explains how the concept of leadership can be framed in a context of public goods games and discusses how to apply evolutionary stability to these games. Section IV presents the main aspects of the model of leadership found in Arce [2001] and discusses some of its shortcomings. Section V generalizes the analysis of the previous section and presents the main results. Section

[1] The issues range from open macroeconomics (Kindleberger [1986]) to environmental problems (Sandler [1993]; Ward [1996]).

[2] See Guttman [1978, 1982].

VI concludes. The proof of Proposition 1 is very long and is presented in the Appendix.

II Evolutionary Stability

The concept of evolutionarily stable strategies (Maynard Smith and Price [1973] and Maynard Smith [1974, 1982]), originally developed in the context of evolutionary biology, tries to capture the notion of a strategy that is robust to evolutionary selection pressures. More specifically, it considers a situation where individuals are repeatedly and randomly drawn from a large population to play a symmetric two-player game. Initially, these individuals are genetically programmed to play a certain strategy (pure or mixed) of this game, but then a small group of individuals, called mutants, who are programmed to play some other strategy, appears in the population. The incumbent strategy is said to be evolutionarily stable if it is able to fend off the assault of any such mutant strategy. More explicitly, there exists an invasion barrier such that, if the share of the population playing the mutant strategy falls below that barrier, the payoff of the incumbent strategy is higher than the payoff of the mutant strategy.

Formally, if we define a symmetric two-player game with strategy space $S=\{1,2,\ldots,n\}$ and payoff function $u:\Delta^2 \to \mathbb{R}$, where $\Delta \equiv \left\{x \in \mathbb{R}_+^n : \sum_{i \in S} x_i = 1\right\}$, then $x \in \Delta$ is an evolutionarily stable strategy (ESS) if, for every $y \in \Delta, y \neq x$, there exists an invasion barrier $\bar{\delta}(y) \in (0,1)$ such that the following condition is satisfied $\forall \delta \in \left(0, \bar{\delta}(y)\right)$:

$$u\left(x, \delta y + (1-\delta)x\right) > u\left(y, \delta y + (1-\delta)x\right) \quad (1)$$

An equivalent way of characterizing an ESS is the following: $x \in \Delta$ is an ESS if, and only if[3]

$$\begin{aligned} &u(x,x) \geq u(y,x) \quad \forall y \in \Delta, \\ &u(x,x) = u(y,x) \Rightarrow u(x,y) > u(y,y) \quad \forall y \neq x. \end{aligned} \quad (2)$$

[3] See, for instance, Weibull [1998].

From the first condition, we can see that an ESS is a best reply against itself. Therefore, every strategy profile (x,x) where x is an ESS is a Nash equilibrium.

There is a "survival of the fittest" flavor to the concept of evolutionary stability. The general idea is that people who use high-payoff strategies have a "reproductive advantage" in comparison to people who use low-payoff strategies. The standard model of this situation in continuous time is the replicator dynamic (Taylor and Jonker [1978]), according to which the rate of growth of a strategy is a linear function of its payoff relative to the average payoff. It doesn't come as a surprise then that every ESS is an asymptotically stable state in the replicator dynamics, as proved by Taylor and Jonker [1978].

A weaker evolutionary stability criterion is that of a neutrally stable strategy. Formally, $x \in \Delta$ is a neutrally stable strategy (NSS) if, for every strategy $y \in \Delta$ there exists some $\bar{\delta}(y) \in (0,1)$ such that $\forall \delta \in \left(0, \bar{\delta}(y)\right)$,

$$u\left(x, \delta y + (1-\delta)x\right) \geq u\left(y, \delta y + (1-\delta)x\right). \tag{3}$$

It is easily seen that, in terms of conditions (2), the only difference between a NSS and an ESS is that the strict inequality in the second condition is replaced by a weak inequality, generating

$$u(x,x) = u(y,x) \Rightarrow u(x,y) \geq u(y,y) \ \ \forall y \in \Delta. \tag{4}$$

III Leadership and Evolution

The problem of how to generate the adequate provision of public goods has been extensively studied under the assumption that altruistic individuals act as leaders and somehow overcome the free rider problem associated with collective decisions. A not so common approach to this problem is to explore the issue of leadership with non-cooperative game theory tools. Arce [2001] suggest the name "leading by example" to describe a process whereby the cooperative solution to a "public good provision" game can be attained through the leader's unilateral commitment to an intermediate level of contribution to the provision of the public good and to matching behavior there beyond.

The phenomenon of leadership is framed in a context of public goods. Let there be *n* agents in the society and let q_i be agent i's contribution to a pure nonexcludable public good. S_i is agent *i*'s strategy space, and hence $q_i \in S_i$. Agent *i*'s preferences are represented by the utility function $V_i : S \to \mathbb{R}$ defined as

$$V_i(q) = U\left(Q(q)\right) - cq_i, \tag{5}$$

where $q = (q_1, \dots, q_n) \in S \equiv S_1 \times S_2 \times \cdots \times S_n$, $Q : S \to \mathbb{R}$ is a function which determines the amount of the public good provided given the individual contributions, and $U : \mathbb{R} \to \mathbb{R}$ is a strictly concave and strictly increasing function.

One can interpret these preferences in the following way. The agent derives utility from the provision of the public good, which depends on the contributions of all agents and is captured by the term $U\left(Q(q)\right)$. On the other hand, there is a cost to contribute to the provision of the public good, which is represented by the linear cost function $C(q_i) = cq_i$.

There are many functional forms that $U(.,.)$ can assume, depending on the way individual contributions are aggregated to produce the public good. The most usual contribution aggregator is the summation aggregator, whose origin may be traced back to Olson's [1965] free rider problem. It assumes that the quantity provided of the public good is equal to the sum of all individual contributions to that good. In this case then, $Q(q) = \sum_{i=1}^{n} q_i$.

There are alternative functional forms *Q* can assume, and they represent different ways public goods aggregate contributions. Hirshleifer [1983,1985] introduced two of them, the weakest-link and best-shot aggregators. The weakest-link aggregator captures the idea that the provision of a public good may be determined by the lowest contribution from members of the collectivity, which in the case of two individuals can be represented by

$$Q = \min\left[q_1, q_2, \dots, q_n\right].$$

The best-shot aggregator bears the opposite idea, that it is the maximum contribution which ultimately determines the amount of public good provided. Formally,

$$Q = \max\left[q_1, q_2, \ldots, q_n\right]$$

Two other important aggregators, weaker-link and better-shot, are less extreme versions of weakest-link and best-shot, respectively. The idea behind weaker-link is that even if one agent is not contributing, it is still possible for the public good to be provided in positive amounts. Cornes [1993] suggests that in this case the amount of public good provided can be represented by

$$Q = \left(\prod_{i=1}^{n} q_i\right)^{\frac{1}{n}},$$

for, given $q = (q_1, q_2, \ldots, q_n)$, lower values of q_i imply higher marginal products[4].

Finally, the better-shot aggregator encompasses the situation where the greatest contribution has the largest marginal effect on the provision of the public good.

As mentioned above, Arce [2001] introduces a new definition of leadership which he calls "leading by example." Leadership is viewed as an additional strategy, and an agent who chooses this strategy acts as a leader, committing himself to a minimum contribution to the provision of the public good and matching higher contributions by other players.

To formalize this notion, we adhere to some of the assumptions of Arce's model. Two agents play a symmetric game where the strategy spaces are the same for both players and given by $S = \{0, 1, 2, q^*\}$. The "leading-by-example" strategy q^* is defined for agent *i* by

$$q^* = \begin{cases} 1, \text{ if } q_j = 0 \text{ or } 1 \\ 2, \text{ if } q_j = 2 \end{cases}, \quad j \neq i. \tag{6}$$

[4] Notice that $\partial Q / \partial q_i = Q / n q_i$, in this case.

The notion of evolutionary stability can now be applied to the normal form game defined by the triple $\left[n = 2, S, \{V_i\}_{i=1,2}\right]$, called the "leading-by-example" game, to test the emergence of leadership as a strategy. The idea is that the selection of q^* by one agent generates a leading behavior which signals to the other agent that coordination aimed at achieving Pareto-superior outcomes is feasible.

In the context of the leading-by-example game, an ESS[5] is defined as a strategy pair (q_i, q_j) which satisfies the conditions:

$$\begin{aligned} & q_i = q_j \\ & V_i(q_i, q_i) \geq V_i(q_i', q_i) \ \ \forall q_i' \neq q_i \\ & V_i(q_i, q_i) = V_i(q_i', q_i) \Rightarrow V_i(q_i, q_i') > V_i(q_i', q_i') \end{aligned} \tag{7}$$

A NSS, in turn, is defined by the same conditions found in (7), with the only difference that the strict inequality in the last condition is replaced by a weak inequality.

IV Leadership when the utility of the public good under the summation aggregator is defined numerically

The main objective of this paper is to identify general conditions on the function U under which the existence of a leader who "leads by example" can entail the cooperative solution. In particular, we want to investigate if Arce's assertion that "cooperative payoff is achieved as a neutrally stable strategy for all leading-by-example games except best shot,[6]" is correct. To do so for all contribution aggregators at once is beyond the scope of this paper, though. We will therefore restrict our attention to the most popular of the aggregators, namely the summation aggregator.

At this point it is important to stress the fact that for other aggregators the distribution of contributions is critical for determining the benefits of collective

[5] We are abusing the notation here, since we are using ESS (and NSS) to denote both strategies and equilibrium pairs.
[6] See Arce[2001, p. 132].

action. However, for the summation aggregator, it does not matter whether one player supplies all of the public good or if its provision is equally divided among the players, as far as benefits go.

In Arce's model, the payoff of player *i* is given by

$$V_i(q_i, q_j) = U(q_i, q_j) - 2q_i,$$

and the summation aggregator is defined numerically, according to the table below:

$U(q_i, q_j)$	q_i	$q_j \le q_i$
0	0	0
5	1	0
11.5	1	1
11.5	2	0
13	2	1
14	2	2

Based on this aggregator, the game in normal form can be represented as below:

	0	**1**	**2**
0	0,0	5,3	11.5,7.5
1	3,5	9.5,9.5	11,9
2	7.5,11.5	9,11	10,10

The free rider problem can be easily identified in this game. In strategy pair $(2,0)$, for instance, player 2 is a free rider, since it contributes nothing and still gets a payoff of 11.5. The Nash equilibria of this game are $(2,0),(0,2)$ and $(1,1)$, while the

Pareto-efficient pairs are $(2,1),(1,2),(2,2),(2,0)$ and $(0,2)$[7]. The only ESS (and NSS) of this game is $(1,1)$, which is not Pareto-efficient.

When the game is extended to include the leading-by-example strategy q^*, it can be represented in normal form as below:

	0	**1**	**2**	q^*
0	0,0	5,3	11.5,7.5	5,3
1	3,5	9.5,9.5	11,9	9.5,9.5
2	7.5,11.5	9,11	10,10	10,10
q^*	3,5	9.5,9.5	10,10	10,10

It can be seen immediately that the Nash equilibria of this game are $(2,0),(0,2),(1,1)$ and (q^*,q^*), of which only (q^*,q^*) is a NSS[8]. Notice that there is no ESS. The Pareto-efficient pairs are $(2,0),(0,2),(2,1),(1,2),(2,2),(q^*,2)$, $(2,q^*)$ and (q^*,q^*).

Arce's conclusion based on the results of the leading-by-example game is that the cooperative outcome can be achieved as a noncooperative evolutionary equilibrium. He is referring to the strategy pair $(2,2)$, which we know is not a NSS, but the same argument is valid for the pair (q^*,q^*), which is associated with the same payoffs as $(2,2)$. It is important to stress though, that the Nash equilibria $(0,2)$ and $(2,0)$ are efficient.

In fact, Arce tries to make an even stronger point, which is summarized by his result number 3: "The cooperative payoff is achieved as an NSS for all leading-by-

[7] Arce[2001] claims that only $(2,1),(1,2)$ and $(2,2)$ are Pareto-efficient strategy profiles

[8] Arce[2001] claims that $(2,2)$ is a NSS, which it cannot be, since it is not even a Nash equilibrium.

example games except best shot, where there was no room for efficiency improvements to begin with.[9]"

V Leadership with a more general definition of the utility of the public good under the summation aggregator

In this section, we generalize the payoff function of the two agents in comparison to Arce's formulation. The generalization is two-fold. First, the marginal cost is now equal to $c > 0$, and not $c = 2$. Second, instead of specifying the image of the function U in (5) numerically, we only require it to be strictly increasing and to satisfy the following condition:

$$U(s+1) - U(s) < U(s) - U(s-1) \quad \forall s \in \{1,2,3\}.^{10} \tag{8}$$

This condition can be interpreted as a diminishing marginal utility or diminishing returns property.

We keep the assumptions of two players and symmetry, as well as the assumption that the strategy space of the extended game is $S = \{0,1,2,q^*\}$.

Since we are dealing with the summation aggregator, we can express

$$U(Q(q_i, q_j)) = f(q_i + q_j). \tag{9}$$

The leading-by-example game in normal form can be represented as in the table below:

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[9] Arce[2001, p.132].

[10] This condition is the analogue to the condition that the second-order derivative of the function U be negative (in the case where U is differentiable), and can be construed as a discrete version of concavity.

	0	**1**	**2**	q^*
0	$f(0), f(0)$	$f(1), f(1)-c$	$f(2), f(2)-2c$	$f(1), f(1)-c$
1	$f(1)-c, f(1)$	$f(2)-c, f(2)-c$	$f(3)-c, f(3)-2c$	$f(2)-c, f(2)-c$
2	$f(2)-2c, f(2)$	$f(3)-2c, f(3)-c$	$f(4)-2c, f(4)-2c$	$f(4)-2c, f(4)-2c$
q^*	$f(1)-c, f(1)$	$f(2)-c, f(2)-c$	$f(4)-2c, f(4)-2c$	$f(4)-2c, f(4)-2c$

We are now ready to offer the proposition below, whose proof can be found in the appendix:

<u>Proposition 1</u>: Consider a "leading-by-example" game $\left[n=2, S, \{V_i\}_{i=1,2}\right]$ *where* $S=\{0,1,2,q^*\}$. *Then:*

(1) q^* can never be an evolutionarily stable strategy;

(2) A necessary condition for q^* to be a neutrally stable strategy is $f(4)-f(1)\geq 2c$ and $f(4)-f(2)\geq c$. A sufficient condition for q^* to be a neutrally stable strategy is $f(4)-f(1)>2c$ and $f(4)-f(2)\geq c$. If $f(4)-f(1)=2c$ and $f(4)-f(2)\geq c$, then (q^*,q^*) is a NSS if, and only if $f(1)\geq c$.

(3) Suppose that $f(1)>c$. Then the strategy $q_i=1$ is evolutionarily stable if, and only if, $f(2)-f(1)\geq c$, $f(3)-f(2)\leq c$ and $f(4)-f(2)<c$. It is neutrally stable if $f(2)-f(1)\geq c$, $f(3)-f(2)\leq c$ and $f(4)-f(2)=c$.

(4) The strategy $q_i=2$ can never be evolutionarily stable. If $f(1)>c$, then it is neutrally stable if $f(2)-f(1)\geq c, f(3)-f(2)\geq c$ and $f(4)-f(3)\geq c$.

There are many important conclusions that can be drawn from this result. First notice though, that we are mostly interested in cases where $f(1) > c$. This makes sense if one is interested in a situation where the benefit of the first unit of the public good is higher that the cost for either agent of contributing to it. As can be checked in the appendix, if we allow $f(1) \leq c$, then there are conditions under which $(0,0)$ can be an ESS and conditions for it to be only a NSS.

The first conclusion that can be drawn is that there are occasions when the evolutionary process will not lead to an equilibrium of this game. If $f(1) > c$, $f(2) - f(1) < c$, $f(4) - f(3) < c$ and $f(4) - f(2) < c$, for instance, then there are no NSS's of this game.

The second conclusion is that the chances of the leading-by-example strategy q^* being a NSS are linked to the "concavity"[11] of the function *U*. In fact, the conditions $f(4) - f(1) \geq 2c$ and $f(4) - f(2) \geq c$ indicate that it is the behavior of the first differences $f(s+1) - f(s)$ of the function *f* (or *U*) that dictates whether or not the leading-by-example strategy is neutrally stable. One possibility is that this function exhibits a high degree of concavity, in the sense that $f(s+1) - f(s)$ is small in comparison to *c* for all *s*, but the conditions in (2) are still satisfied because the first differences are not much smaller than *c*. Another possibility is that the function has a high degree of concavity, in the sense that the first differences are larger than *c*, and then q^* is automatically sustained as a neutrally stable strategy[12]. This result makes sense, since when the marginal utility of the consumption of the public good does not decrease at a too high a rate, the gains the society obtains when larger contributions to the provision of the public good are made are higher, increasing the chances of success of the leader.

A third and more obvious conclusion is that the higher the cost of contributing to the provision of the public good, the smaller the chances of reaching equilibria with high levels of contributions in an evolutionary process. In fact, if $f(1) < c$, it is possible for the only outcome of the evolutionary process to be zero contribution by

[11] Remember that we cannot strictly talk about concavity since the function *U* is defined on a discrete domain.
[12] The proof of proposition 1 in the appendix explores these two possibilities.

both players. Moreover, when c is high, it is harder to satisfy the conditions for q^* and *2* to be neutrally or evolutionarily stable strategies.

Finally, it is important to investigate the implications of Proposition 1 to the chances of success of cooperation in an evolutionary process. We will do that next for a particular case, namely the one where $f(1) > c$ and $f(2) - f(1) < c$. The reason for this choice is that, as can be verified in the proof of Proposition 1 in the Appendix, the other cases are not very interesting in the sense that there are a lot of Nash equilibria, some of them already Pareto-efficient. The results obtained by Arce [2001], for instance, are obtained when a particular case of $f(1) > c, f(2) - f(1) \geq c$ and $f(3) - f(2) < c$ is considered, and we've seen above that the Nash equilibria $(0,2)$ and $(2,0)$ are efficient in this case.

In order to guarantee that (q^*, q^*) is an NSS of the extended (leading-by-example) game, we will add one assumption to $f(1) > c$ and $f(2) - f(1) < c$, namely $f(4) - f(1) > 2c$. In this case, the Nash equilibria of the leading-by-example game are $(1,0), (0,1)$ and (q^*, q^*).

Let's turn first to the original game, where there is no strategy q^*. The following proposition identifies the Pareto-efficient strategy pairs under the assumptions above.

Proposition 2: When the conditions $f(1) > c$, $f(2) - f(1) < c$ *and* $f(4) - f(1) > 2c$ *are satisfied, the Pareto-efficient strategy pairs of the game* $\left[n = 2, S, \{V_i\}_{i=1,2}\right]$, *where* $S = \{0,1,2\}$, *are* $(2,0), (0,2)$, $(2,1), (1,2)$ and $(2,2)$.

Proof: We need to test all strategy pairs for Pareto-efficiency.

Since $f(4) - f(1) > 2c$, we have that

$$f(4) - 2c > f(1) > f(1) - c,$$

and hence $(1,0)$ and $(0,1)$ are Pareto-dominated by $(2,2)$.

It is trivial to show that $(0,0)$ is Pareto-dominated by $(2,2)$. It is also easy to show that $(1,1)$ is Pareto-dominated by $(2,2)$. In fact, conditions $f(4)-f(1)>2c$ and $f(2)-f(1)<c$ together imply

$$\begin{aligned}&f(4)-f(2)=f(4)-f(1)+f(1)-f(2)>2c-c=c\\&\Rightarrow f(4)-2c>f(2)+c-2c=f(2)-c.\end{aligned}$$

Comparing now $(1,2)$ with $(2,2)$, we can observe that $f(4)-2c>f(3)-2c$, for *f* is a strictly increasing function. On the other hand, $f(4)-f(3)<c$ implies

$$f(4)-2c=f(4)-f(3)+f(3)-2c<c+f(3)-2c=f(3)-c.$$

Therefore, there is no relation of dominance between $(1,2)$ (or $(2,1)$) and $(2,2)$.

The next comparison we have to make is between $(2,0)$ and $(2,2)$. Notice that

$$f(2)-2c<f(4)-2c$$
and
$$f(4)-f(2)<2c\Rightarrow f(4)-2c<f(2),$$

which means that there is no relation of dominance between these two strategy pairs (and no dominance between $(0,2)$ and $(2,2)$ either).

The conclusion we can reach at this point is that $(2,2)$ is Pareto-efficient. It remains t test whether or not $(2,0)$ and $(1,2)$ are Pareto-efficient.

With respect to $(2,0)$, we can easily show that it is not Pareto-dominated by $(0,0),(1,0),(0,1),(1,1),(0,2)$ or $(2,2)$. Besides, since $f(3)-f(2)<c$ implies $f(2)>f(3)-c$, $(2,0)$ is not Pareto-dominated neither by $(2,1)$ nor by $(1,2)$. Therefore, $(2,0)$ and $(0,2)$ are Pareto-efficient. A similar argument shows that $(1,2)$ and $(2,1)$ are also Pareto-efficient.

As a matter of fact, since, on one hand, $(2,2)$ Pareto-dominates $(0,0),(1,0),(0,1)$ and $(1,1)$, and, on the other hand, $(2,0)$ and $(1,2)$ are not Pareto-dominated by $(2,2)$, we can conclude directly that $(0,0),(1,0),(0,1)$ and $(1,1)$ cannot Pareto-dominate $(2,0)$ and $(1,2)$. To see this, suppose $(2,0)$ was Pareto-dominated by $(1,0)$, for instance. Then it would be possible to increase the payoff of one of the agents without reducing the other's by changing from pair $(2,0)$ to pair $(1,0)$. But since $(1,0)$ is dominated by $(2,2)$, it would be possible to raise the payoff of one of the agents without reducing the other's by moving from $(2,0)$ to $(2,2)$, which is a contradiction.

Therefore, we conclude that the Pareto-efficient strategy pairs are $(2,0),(0,2)$, $(2,1),(1,2)$ and $(2,2)$. ■

It can be easily verified that the only Nash equilibria of the original game when the conditions of Proposition 2 are satisfied are $(0,1)$ and $(1,0)$, none of which can be ESS or NSS. This means that, in the absence of the leading-by-example strategy, it is not possible to reach Pareto-efficient outcomes in an evolutionary process (as defined here).

If we keep the assumptions of Proposition 2 but go back to the leading-by-example game, where $S=\{0,1,2,q^*\}$, we can easily verify that the Nash equilibria are $(1,0),(0,1)$ and (q^*,q^*). We therefore have the following proposition:

Proposition 3: If the conditions of Proposition 2 are satisfied, then (q^*,q^*) *is a NSS of the leading-by-example game and is Pareto-efficient.*

Proof: Notice that $f(4)-f(1)>2c$ and $f(2)-f(1)<c$ imply

$$f(4)-f(2)=f(4)-f(1)+f(1)-f(2)>2c-c=c$$

Since $f(1)>c$, Proposition 1 tells us that (q^*,q^*) is an NSS.

Regarding efficiency, we can easily show that, since $V_i\left(q^*,q^*\right)=V_i\left(2,2\right)$, $\left(q^*,q^*\right)$ is Pareto-efficient. ■

What this result tells us is that, even if the marginal utility of the consumption of the public good decreases at a relatively fast pace (notice that, in the case we are considering, $f\left(1\right)>c$ but $f\left(4\right)-f\left(3\right)<f\left(3\right)-f\left(2\right)<f\left(2\right)-f\left(1\right)<c$) and Nash equilibria are not Pareto-efficient in the absence of a leader who leads by example, it is possible to achieve the cooperative (Pareto-efficient) outcome in an evolutionary process if the agents are allowed to follow a leading-by-example strategy.

It is important to notice though, that if we replaced the assumption $f\left(4\right)-f\left(1\right)>2c$ in Proposition 2 by $f\left(4\right)-f\left(1\right)<2c$ or $f\left(4\right)-f\left(2\right)<c$, keeping the other two, the existence of a leading-by-example strategy would not steer the society to a Pareto-efficient outcome in an evolutionary process, for the simple reason that there would be no evolutionarily or neutrally stable strategies of the extended game.

One last comment is that, even if there are no Pareto-efficient ESS or NSS in a game, it may happen that some of the Nash equilibria are Pareto-efficient. In particular, there are many games where free-riding Nash equilibria are Pareto-efficient, like Arce's game discussed in section IV.

VI Conclusion

As we mentioned in the text, one of the main contributions of Arce [2001] is his point that the cooperative payoff is achieved as a neutrally stable strategies equilibrium for all leading-by-example games except best shot. In this paper, we concentrated on evaluating general conditions under which this assertion is valid when the summation aggregator is used to determine the level of provision of the public good.

Our first conclusion is that it is not always true that the cooperative payoff can be achieved in a leading-by-example game for the summation aggregator. We obtained conditions under which there doesn't even exist any neutrally stable strategy of this game. These conditions, and that is the second contribution of this paper, are closely associated with what we called the degree of concavity of the utility function valuing the consumption of the public good. We did not define this degree of concavity precisely because we are dealing with discrete domains.

We also showed that when this function has a high degree of concavity, in the sense that the first differences are larger than the cost of contributing, then the leading-by-example strategy can be sustained as a neutrally stable strategy. This result is in accordance with our expectations, since when the marginal utility of the consumption of the public good does not decrease at a steep rate, the gains the society obtains from larger contributions to the provision of the public good are higher, increasing the chances of success of the leader. It is important to mention that we should be able to generalize this result to continuous settings, pending upon a good definition of leadership in such a context.

Finally, we were able to demonstrate the possibility of generating Pareto-efficient outcomes as the result of evolutionarily stable processes even when the marginal utility of the consumption of the public good decreased at a relatively fast pace and Nash equilibria were not Pareto-efficient in the absence of a leader who leads by example. This step was necessary because the existence of evolutionarily or neutrally stable strategies does not immediately guarantee they will be Pareto-efficient.

It is important to stress that the scope of this paper is very limited and that the results presented here are very preliminary. There are many directions in which they can be extended. A first direction was already mentioned above, namely the generalization to continuous settings. An obvious extension is to apply a similar method to other contribution aggregators, such as weakest-link, weaker-link, best-shot and better-shot. Another extension worth mentioning is to try different varieties of learning behavior against this leadership background. There are undoubtedly many others.

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VIII Appendix

Proof of Propostion 1:

We will prove this result by identifying all the Nash equilibria and all evolutionarily and neutrally stable strategies, although there is a much shorter proof. The reason for choosing the longer path is that the identification of all the Nash equilibria, ESS and NSS will prove to be useful later on.

First of all, we need to establish, for each strategy profile, the necessary and sufficient conditions for it to be a Nash equilibrium. These conditions are presented in the table below:

Profile	***Conditions***
$(0,0)$	$f(1)\leq c, f(2)\leq 2c$
$(1,0)$	$f(1)\geq c, f(2)-f(1)\leq c, f(3)-f(1)\leq 2c$
$(2,0)$	$f(2)\geq 2c, f(2)-f(1)\geq c, f(3)-f(2)\leq c, f(4)-f(2)\leq 2c$
$(q^*,0)$	$f(1)\geq c, f(2)-f(1)\leq c, f(4)-f(1)\leq 2c$

$(0,1)$	$f(1) \geq c, f(2)-f(1) \leq c, f(3)-f(1) \leq 2c$
$(1,1)$	$f(2)-f(1) \geq c, f(3)-f(2) \leq c$
$(2,1)$	$f(3)-f(1) \geq 2c, f(3)-f(2) \geq c, f(4)-f(3) \leq c$
$(q^*,1)$	$f(2)-f(1) \geq c, f(3)-f(2) \leq c, f(4)-f(2) \leq c$
$(0,2)$	$f(2) \geq 2c, f(2)-f(1) \geq c, f(3)-f(2) \leq c, f(4)-f(2) \leq 2c$
$(1,2)$	$f(3)-f(1) \geq 2c, f(3)-f(2) \geq c, f(4)-f(3) \leq c$
$(2,2)$	$f(4)-f(2) \geq 2c, f(4)-f(3) \geq c$
$(q^*,2)$	$f(4)-f(2) \geq 2c, f(4)-f(3) \geq c, f(4)-f(1) \geq 2c$
$(0,q^*)$	$f(1) \geq c, f(2)-f(1) \leq c, f(4)-f(1) \leq 2c$
$(1,q^*)$	$f(2)-f(1) \geq c, f(3)-f(2) \leq c, f(4)-f(2) \leq c$
$(2,q^*)$	$f(4)-f(2) \geq 2c, f(4)-f(3) \geq c, f(4)-f(1) \geq 2c$
(q^*,q^*)	$f(4)-f(1) \geq 2c, f(4)-f(2) \geq c$

Notice that the conditions for (q_i, q_j) to be an equilibrium are the same for (q_j, q_i) to be an equilibrium.

We consider now several cases.

Case 1: $f(1) \leq c$

In this case, (8) implies

$$f(4)-f(3) < f(3)-f(2) < f(2)-f(1) < f(1) \leq c$$

and

$$f(2) = f(2)-f(1)+f(1) < 2c.$$

Therefore,

$$\begin{aligned}
&f(4)-f(1)=f(4)-f(3)+f(3)-f(2)+f(2)-f(1)<3c\\
&f(4)-f(2)=f(4)-f(3)+f(3)-f(2)<2c\\
&f(3)-f(1)=f(3)-f(2)+f(2)-f(1)<2c
\end{aligned}$$

We conclude that the possible Nash equilibria in this case are: $(0,0),(1,0)$ (as long as $f(1)=c$), $(0,1)$[13], $(q^*,0)$ (as long as $f(1)=c$ and $f(4)-f(1)\leq 2c$), $(0,q^*)$ and (q^*,q^*) (as long as $f(4)-f(1)\geq 2c$ and $f(4)-f(2)\geq c$).[14]

Since any ESS or NSS has to be a symmetric pair of strategies, the only candidates among the Nash equilibria are $(0,0)$ and (q^*,q^*). Let's first take a look at $(0,0)$. We already now that $f(2)<2c$, which means that $V_i(0,0)>V_i(2,0)$. We also now that $f(1)\leq c$. If $f(1)<c$, then $V_i(0,0)>V_i(1,0)=V_i(q^*,0)$ and we conclude that $(0,0)$ is an ESS. If $f(1)=c$, then $V_i(0,0)=V_i(1,0)=V_i(q^*,0)$. Since $V_i(0,1)=f(1)>f(2)-c=V_i(1,1)$, $V_i(0,q^*)=f(1)$ and $V_i(q^*,q^*)=f(4)-2c$, we conclude that $(0,0)$ is an ESS if $f(4)-f(1)<2c$ and a NSS if $f(4)-f(1)=2c$.

Now let's turn to (q^*,q^*). Notice that $V_i(q^*,q^*)=V_i(2,q^*)$, $i=1,2$, which means that we have to compare $V_i(q^*,2)$ and $V_i(2,2)$. Since they are equal, we conclude that (q^*,q^*) can only be a NSS, not an ESS. Notice that this argument is general, i.e., it is valid whenever the conditions for (q^*,q^*) to be a Nash equilibria are satisfied. But what happens if $f(4)-f(1)=2c$ or $f(4)-f(2)=c$. In the first case, $V_i(q^*,q^*)=V_i(0,q^*)$, and we thus have to compare $V_i(q^*,0)=f(1)-c$ with $V_i(0,0)=f(0)$. Since $f(1)\leq c$, we conclude that $V_i(q^*,0)\leq V_i(0,0)$, and thus (q^*,q^*) can only be a NSS when $f(1)=c$. In the second case, $V_i(q^*,q^*)=V_i(1,q^*)$,

[13] Remember that the game is symmetric.

[14] Notice that $(q^*,0),(0,q^*)$ and (q^*,q^*) can all be Nash equilibria at the same time only if $f(4)-f(1)=2c$.

and we have to compare $V_i(q^*,1)$ with $V_i(1,1)$. Since they are equal, we conclude that (q^*,q^*) is a NSS in this case. In summary, if $f(4)-f(1)>2c$ and $f(4)-f(2)\geq c$, then (q^*,q^*) is a NSS. If $f(4)-f(1)=2c$ and $f(4)-f(2)\geq c$, then (q^*,q^*) is a NSS when $f(1)=c$ and is not a NSS when $f(1)<c$.

Case 2: $f(1)>c$. We will divide this case in several sub cases.

(i) $f(2)-f(1)<c$

In this sub case, the following inequalities hold:

$f(4)-f(3)<f(3)-f(2)<f(2)-f(1)<c,$

and this implies

$$f(4)-f(1)<3c, f(4)-f(2)<2c, f(3)-f(1)<2c.$$

Therefore, the possible Nash equilibria are: $(1,0)$, $(q^*,0)$ (as long as $f(4)-f(1)\leq 2c$), $(0,1),(0,q^*)$ and (q^*,q^*) (as long as $f(4)-f(1)\geq 2c$ and $f(4)-f(2)\geq c$).

To determine whether or not (q^*,q^*) is a NSS, we have to examine the cases $f(4)-f(1)=2c$ and $f(4)-f(2)=c$. Notice that now $f(1)>c$, and thus $V_i(q^*,0)>V_i(0,0)$, which makes the first case irrelevant. In the second case, as we know, $V_i(q^*,q^*)=V_i(1,q^*)$ and $V_i(q^*,1)=V_i(1,1)$. Hence we conclude that (q^*,q^*) is in fact a NSS.

(ii) $f(2)-f(1)\geq c, f(3)-f(2)<c$

In this sub case, the following inequalities hold:

$$f(4)-f(3)<f(3)-f(2)<c,$$

which implies

$$f(4) - f(2) < 2c.$$

Based on these conditions, we conclude that the possible Nash equilibria are: $(1,0)$ (as long as $f(2) - f(1) = c$), $(2,0)$ (as long as $f(2) \geq 2c$), $(q^*,0)$ (as long as $f(2) - f(1) = c$ and $f(4) - f(1) \leq 2c$), $(0,1)$, $(1,1)$, $(q^*,1)$ (as long as $f(4) - f(2) \leq c$), $(0,2)$, $(0,q^*)$, $(1,q^*)$ and (q^*,q^*) (as long as $f(4) - f(1) \geq 2c$ and $f(4) - f(2) \geq c$).

Now there are two candidates for ESS and NSS, namely $(1,1)$ and (q^*,q^*). We can use the same argument as in 2(i) to conclude that (q^*,q^*) is a NSS but not an ESS when $f(4) - f(1) \geq 2c$ and $f(4) - f(2) \geq c$.

Regarding $(1,1)$, notice that $V_i(1,1) = V_i(q^*,1)$. Since

$$V_i(1,q^*) = f(2) - c \text{ and } V_i(q^*,q^*) = f(4) - 2c$$

we conclude that if $f(4) - f(2) < c$, then $(1,1)$ can be an ESS, and if $f(4) - f(2) = c$, then it can be an NSS.

If $f(2) - f(1) = c$, then $V_i(1,1) = V_i(0,1)$. But since $V_i(1,0) = f(1) - c$ and $V_i(0,0) = f(0)$, we have $V_i(1,0) > V_i(0,0)$. Finally, since $f(3) - f(2) < c$, we have $V_i(1,1) > V_i(2,1)$. Notice that if $f(3) - f(2) = c$, then $V_i(1,1) = V_i(2,1)$ but

$$V_i(1,2) = f(3) - c > f(4) - 2c = V_i(2,2).$$

In summary, we have that, if $f(4) - f(1) \geq 2c$ and $f(4) - f(2) > c$, then only (q^*,q^*) is a NSS. If $f(4) - f(1) \geq 2c$ and $f(4) - f(2) = c$, then both (q^*,q^*) and $(1,1)$ are NSS's. If $f(4) - f(2) < c$, then $(1,1)$ is an ESS but (q^*,q^*) is not even a Nash equilibrium.

(iii) $f(2) - f(1) \geq c, f(3) - f(2) \geq c, f(4) - f(3) < c$

The possible Nash equilibria in this case are: $(1,0)$ (as long as $f(2)-f(1)=f(3)-f(2)=c$), $(2,0)$ (as long as $f(3)-f(2)=c$), $(q^*,0)$ (as long as $f(2)-f(1)=c$ and $f(4)-f(1)\leq 2c$), $(0,1)$, $(1,1)$ (as long as $f(3)-f(2)=c$), $(2,1)$ (as long as $f(3)-f(1)\geq 2c$), $(0,2)$, $(1,2)$, $(0,q^*)$ and (q^*,q^*).

Notice that the conditions for (q^*,q^*) to be an equilibrium are satisfied in this case, since

$$f(4)-f(1)=f(4)-f(3)+f(3)-f(2)+f(2)-f(1)\geq f(4)-f(3)+2c>2c$$
$$f(4)-f(2)=f(4)-f(3)+f(3)-f(2)\geq f(4)-f(3)+c>c.$$

It is also easy to see that $(q^*,1)$ and $(1,q^*)$ cannot be Nash equilibria. In fact, for that to happen it would be necessary that $f(3)-f(2)=c$ and $f(4)-f(2)\leq c$, which is impossible, since

$$f(4)-f(2)=f(4)-f(3)+f(3)-f(2)=f(4)-f(3)+c\leq c$$

can only happen if $f(4)-f(3)\leq 0$.

Since (q^*,q^*) is a Nash equilibrium, we already now that it is a NSS but not an ESS. We also know that $f(4)-f(2)>c$, and hence, according to our previous reasoning, $(1,1)$ is neither an ESS nor a NSS.

(iv) $f(2)-f(1)\geq c, f(3)-f(2)\geq c, f(4)-f(3)\geq c$

The conditions above imply

$$f(4)-f(2)\geq 2c, f(4)-f(1)\geq 3c, f(3)-f(1)\geq 2c$$

Therefore, the possible Nash equilibria are: $(1,0)$ (as long as $f(2)-f(1)=f(3)-f(2)=c$), $(2,0)$ (as long as $f(3)-f(2)=f(4)-f(3)=c$), $(0,1)$, $(1,1)$ (as long as $f(3)-f(2)=c$), $(2,1)$ (as long as $f(4)-f(3)=c$), $(0,2)$, $(1,2)$, $(2,2)$, $(q^*,2)$, $(2,q^*)$ and (q^*,q^*).

Among the Nash equilibria, only $(1,1),(2,2)$ and (q^*,q^*) can be ESS or NSS. A reasoning similar to that presented in sub case 2(iii) indicates that (q^*,q^*) is a NSS but that $(1,1)$ is neither an ESS nor a NSS.

Regarding $(2,2)$, we know that $V_i(2,2)=V_i(q^*,2)$ and that $V_i(2,q^*)=V_i(q^*,q^*)$. Now if $f(4)-f(3)=c$, then $V_i(2,2)=V_i(1,2)$. Since $V_i(2,1)=f(3)-2c$, $V_i(1,1)=f(2)-c$ and $f(3)-f(2)\geq c$, we have that $V_i(2,1)\geq V_i(1,1)$. Moreover, if $f(4)-f(2)=2c$, then $V_i(2,2)=V_i(0,2)$. Since $V_i(2,0)=f(2)-2c$, $V_i(0,0)=f(0)$, $f(2)-f(1)\geq c$ and $f(1)>c$, we have that $V_i(2,0)>V_i(0,0)$. Therefore, $(2,2)$ is a NSS but not an ESS. ■

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