



Universidade de Brasília
Departamento de Economia

Série Textos para Discussão

**Structural Analysis of Multiple-Unit Auctions:
Recovering Bidders' Valuations in Auctions with
Dominant Bidders**

Anderson Caputo Silva
Secretaria do Tesouro Nacional

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Structural Analysis of Multiple-Unit Auctions: Recovering bidders' valuations in auctions with dominant bidders*

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October, 2002

Abstract

Recent work has attempted to develop structural analysis for empirical study of multi-unit auctions. Hortaçsu (2000) is the first to build a model that is both consistent with the strategic divisible good framework and computationally tractable. However, Hortaçsu's methodology appears to provide limited results in auctions which some bidders bid for large shares of the amount being auctioned. I develop an alternative methodology that distinguishes between dominant and non-dominant bidders. I use this methodology to examine Brazilian treasury auctions and show that it provides improved results. As an example of possible uses for this methodology I investigate heterogeneity across different categories of bidders and find evidence that suggests the importance of private components in bidders' valuations in Brazilian auctions. More specifically, national commercial banks seem to show higher elasticity of demand compared to investment banks and brokers. This result indicates that caution should be exercised in studies that assume that treasury auctions are in line with a pure common values setting.

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1. Introduction

Recovering the actual valuation of bidders for goods that are sold through multiple-price multiple-unit auctions has been a major challenge to the auction literature. Given that these valuations are not directly observable, auction researchers depend on the development of strategic bidding models that allow the estimation of such valuations based on the bids presented in the auction.

Unfortunately, modeling strategic bidding in multiple-unit auctions is an awkward task both analytically and computationally. Despite the extensive literature, very few studies have adopted a structural approach to the empirical examination of this type of auctions. In addition, among the few studies that apply structural analysis in multiple-unit auctions, only Hortaçsu (2000) is in fact consistent with a strategic divisible good auction framework. The lack of a theoretical model that encompasses the complexities of these auctions allied to the difficult access to data at the individual bidding level, has led most researchers to use reduced form models that take bids as the variable to be explained.

The scope of issues that can be addressed through reduced form models, however, is very limited if compared to that of structural models. Structural models have the important advantage of being directly derived from an underlying auction model which allows the testing of the model itself and not only some of its implications. Another advantage of structural models is their ability to predict bids in case of a change in structure, such as the introduction of a subsidy or tax on the buyer side. Since such changes will affect the distribution of bidders' valuations, the actual bids can be predicted only with a structural model.¹

The few studies that apply structural analysis in multi-unit auctions, especially those related to auctions of government securities, have focused on applications related to improved choice of auction mechanism. Hortaçsu (2000) and Heller and Lengwiler (1998), for example, examine the auction format (uniform or discriminatory price) that would yield higher revenues in auctions of Turkish and Swiss government securities, respectively.

Although the methodology for recovering bidders' valuations that Hortaçsu suggests is so far the only one that is consistent with a multi-unit auction framework, I show that it provides quite limited results in auctions that individual bidders are allowed to buy significant shares of the total amount being auctioned. As described in more detail below, under Hortaçsu's approach dominant bidders (those who bid for large quantities) distort the estimation of marginal valuations.

¹ See Laffont, Ossard and Vuong (1995) for these arguments on the context of auctions.

The objective of the paper is to propose an empirical methodology that, based on theoretical foundations suggested by Hortaçsu (2000), allows the estimation of bidders' marginal valuations in auctions with dominant bidders. Using data from auctions of Brazilian government securities I show that this modified methodology provides improved estimates of unobservable valuations. An important contribution of this paper is that it is the first to investigate differences in bidders' valuations across distinct categories of bidders based on a model that estimates the unobservable valuation of auction participants.

In order to illustrate the type of issues that can be addressed with the use of this structural analysis tool, I investigate whether different categories of bidders are heterogeneous with respect to their demand schedules (price-quantity bid vectors). More specifically, I compute the elasticity of demand of each bid vector submitted by auction participants and find systematic differences in the elasticity of demand across bidders' categories. To the extent that most studies in the empirical literature of government securities conduct tests of auction literature predictions assuming a common values setting with symmetrically informed bidders, this result indicates that caution should be exercised in their interpretation.

The paper is organized as follows: Section 2 describes the methodology that I propose to recover bidders' valuations in auctions with dominant bidders. Using data from 6 auctions of Brazilian treasury bills, I employ the proposed methodology in section 3. The estimates of bidders' marginal valuations obtained in this exercise allows me to compute the demand elasticity of each vector of bids presented by individual bidders. Classifying bidders into brokers, commercial or investment banks (and into national or foreign banks), I examine in section 4 whether there are systematic differences in demand elasticities across these categories of bidders.

2. Structural estimation in the presence of dominant bidders

The empirical methodology proposed in this paper is based on Hortaçsu (2000). Using as guideline Wilson (1979) and Nautz (1995), Hortaçsu models strategic behavior in a discriminatory auction and proposes an estimation method to reconstruct the distribution of marginal valuations. Besides reducing the high computational demands usually required in the analysis of structural models, Hortaçsu's work has the advantage of being the first empirical paper in the literature on auctions whose theoretical foundations are consistent with a strategic divisible good auction framework. The following subsection provides a brief description of Hortaçsu's model and his empirical methodology.

2.1. Hortaçsu's (2000) model of bidder behavior

Let Q denote the total supply in an auction with N participating bidders (where N is common knowledge). Given the set of possible prices $p_0 < p_1 < \dots < p_{K+1}$ on a finite grid, each bidder submits a bid vector y_i defined as quantities specified for each of these prices, i.e.: $y_i : \{y_{i0} \geq y_{i1} \geq \dots \geq y_{iK+1}\}$.

The market clearing price, p_{k^*} , is determined after sorting all price-quantity bids (in decreasing order of prices) and finding the price at which total demand just falls short of the total supply, i.e.:

$$p_k : k^* = \min \left\{ k : \sum_{i=1}^N y_{ik} \leq Q \right\}$$

Hortaçsu assumes that, for y units of the security, the marginal valuation of each bidder is given by $v_i(y, s_i, W)$ which is a function of a set of publicly observable economic variables W before the auction, and a private signal component, s_i . One of the main assumptions of the paper is that private signals are independent, conditional on W . Assuming independence of private signals guarantees the existence of an equilibrium to the game and makes the setup described above one of independent private values.

The author defines $H(p_k, y_i)$ to be the probability that the market clearing price is below p_k conditional on the bid vector of bidder i , y_i , i.e.:

$$H(p_k, y_i) = \Pr\{\text{market clearing price} \leq p_k, \text{ given } y_i\}$$

Given this set up Hortaçsu constructs the expected payoff function of a risk neutral bidder and derive the following first order condition:

$$v_i(y_{ik}, s_i) = p_k + \frac{H(p_{k-1}, y_i)(p_k - p_{k-1})}{H(p_k, y_i) - H(p_{k-1}, y_i)}$$

Note that according to the expression above, by estimating $H(p_k, y_i)$ and $H(p_{k-1}, y_i)$ one may reconstruct the marginal valuation that rationalizes a bid of p_k for y_{ik} units of the security.² Following the steps described below, Hortaçsu proposes a methodology to estimate $H(p_k, y_i)$:

1. Fix bidder i among the total N bidders in the auction.
2. From the sample of remaining $(N - 1)$ bid vectors, draw a random sample of $(N - 1)$ bid vectors with replacement, giving equal probability of $1/(N-1)$ to each bid vector in the original sample.
3. Construct the residual supply function generated by these $N - 1$ “resampled” bid vectors.
4. Intersect with bidder i ’s bid to find the market clearing price.
5. Repeat B times for each bidder and for all bidders in the data set.

This generates B market clearing prices conditional on y_i . Hortaçsu then estimates $H(p_k, y_i)$ by counting the frequency which a given p_k remained above the market clearing prices generated above. The author calls the above estimator of $H(p_k, y_i)$, $H^R(p_k, y_i)$, the resampling estimator of the probability distribution of the market clearing price and shows that it is a consistent estimator.

Although the author successfully employs his methodology using data from Turkish treasury auctions, I obtain less satisfactory results with Brazilian data. In Brazil the individual demand of some (dominant) bidders frequently represents a significant share of the total amount supplied in the auction. This fact, as described in the following subsection, imposes an important restriction to the estimation of the marginal valuations of smaller bidders.

2.2 Adapting the methodology to the existence of dominant bidders.

Note that the proposed method to estimate $H(p_k, y_i)$ and $H(p_{k-1}, y_i)$ consists of building a distribution of residual supply functions obtained through the B sets of random draws from the empirical distribution of bid vectors observed in the data. The problem that a dominant bid vector causes to such procedure is that of turning the residual supply functions very flat. In other words, the probability distribution of winning the auction at each given price is highly influenced by the existence of dominant price-quantity vectors.

² The lowest bid of each bid vector is assumed to have marginal valuation identical to its corresponding price, i.e:

$$v_i(y_{ik}^1, s_i) = p_k^1$$

In order to illustrate the argument above, I apply Hortaçsu's procedure to an auction of 3-month Brazilian fixed-rate securities, the so-called LTN, held by the treasury in January of 1998. As typical in these auctions, one bidder (henceforth, bidder #1) submitted a vector of bids that accounted for over 20% of the total amount supplied. More specifically, bidder #1's total demand reached 44.4% of the R\$ 2.5 billion being auctioned, whereas the demand of each of the 51 remaining bidders corresponded on average to 3.8%. Table 1 below provides summary statistics on the mentioned auction.

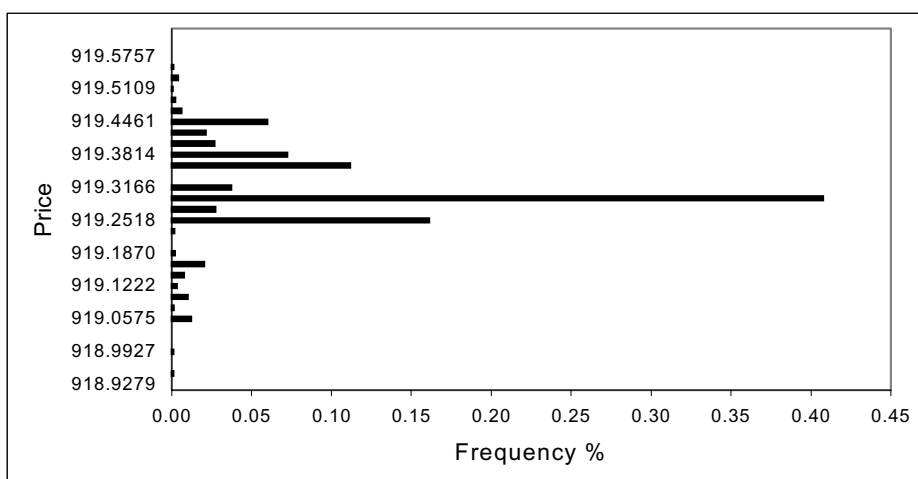
Table 1 – Auction of 3-month Brazilian T-bills (LTN)* in Jan/06/98

	Total
Amount auctioned (in # of T-bills)	2,500,000
Total Demand (in # of T-bills)	5,977,500
Non-dominant bidders' demand (in # of T-bills)	4,868,500
Dominant bidder's demand (in # of T-bills)	1,109,000
Number of bidders	52
Number of bids	171
Cutoff Price	919.2830

*Face Value = R\$ 1,000

Fixing bidder #2 among the total of 52 bidders, I draw 10,000 random samples of 51 bid vectors each. I then construct (10,000) residual supply functions implied by these resampled bid vectors and intersect with bidder #2's bids to find cutoff prices. Graph 1 shows the distribution of cutoff prices that I obtain with this procedure.

Graph 1 – Distribution of cutoff prices



The evident peak in the distribution, accounting for approximately 41% of the cutoff prices, is represented by prices from 919.2734 up to 919.2950. In fact, taking a closer look at this interval, I find that the cutoff price is exactly 919.2830 in 3,801 out of the 10,000 simulations conducted. Note also that this is the same cutoff price as the one observed in the “real” auction. This result indicates that whenever the bootstrap procedure randomly selects the dominant bid vector from the sample of bid vectors submitted in the auction, the distribution of cutoff prices tends to be too concentrated at certain price levels and flat for prices further apart.

Recall that in Hortaçsu’s first order condition, shown previously, the denominator of the “shading factor” represents the difference in the probabilities of winning the auction at prices p_k and p_{k-1} . Therefore, in case these prices are located far from the point where most of the mass of the distribution is concentrated, the denominator tends to zero and the estimated valuation that would be consistent for each of those prices goes to infinity. This is what actually occurs in the auction examined above. Taking the 5 bids presented by bidder #2 as an example, I report in table 2 the estimates of marginal valuations obtained with Hortaçsu’s methodology. Note that at least two of bidder #2’ s valuations present unrealistic levels, driven by the extremely high probability of winning at any of her three highest bids.

Table 2 – Estimated Marginal Valuations implicit in Bidder #2’s bids

Price bid	$H^R(p_k, y)$	Valuation
919.2820	0.2631	919.2820
919.4335	0.9361	919.4927
919.5851	0.9999	921.8094
919.7366	1	2434.585
919.8882	1	(infinite)

The example above points out the problem of using realized data to build up a distribution of residual supply functions that bidders expected to face in the auction. Upon presenting bids at prices 919.7366 and 919.8882, for example, bidder #2 may have inferred that the probability of winning at each of those prices was significantly different to justify such price distinction. Hortaçsu’s bootstrap methodology works well in auctions which all bidders bid for similar quantities. However, the presence of dominant bidders distorts the distribution of cutoff prices, making it too dependent on the actual bid vector submitted by this type of bidders.

Since the price bid by any particular bidder is supposedly a random variable from the point of view of all other bidders, a proposed solution to the presence of dominant bidders would be to treat their bid vectors as realizations drawn from a distribution of dominant price-quantity vectors. To do this requires a definition of dominant bids and a decision about modeling their distribution. Table 3 shows

summary statistics of the ratio of total demand to total supply corresponding to 3,795 bid vectors submitted in 83 Brazilian T-bill auctions held from January 1998 to June 1999.

Table 3 – Frequency distribution bid vector demand/auction supply
(3,795 bid vectors)

Bin	Frequency	Cumulative %
1%	809	21.32%
5%	1591	63.24%
10%	712	82.00%
15%	281	89.41%
20%	174	93.99%
25%	65	95.70%
30%	54	97.13%
35%	24	97.76%
40%	27	98.47%
45%	10	98.74%
50%	15	99.13%
60%	6	99.29%
70%	4	99.39%
80%	5	99.53%
90%	1	99.55%
More	17	100.00%

Note that approximately 63% of the bid vectors submitted in this sample of Brazilian auctions have total demand that account for 5 % or less of the amount auctioned. Although the exact definition of a dominant bid vector remains subjective, table 2 provides a good illustration of the relevance of treating some bid vectors as a separate group.

In the methodology I describe in more detail below, I consider as dominant bid vectors those which their total demand correspond to more than 20% of the supply. I obtain a sample of 310 dominant bid vectors and compute the average price corresponding to each of those vectors. The resulting prices, denoted P_{dom} , are regressed against variables accounting for the expected resale price of the security being auctioned, the expected average price bid by non-dominant bidders, the expected demand of non-dominant bidders, and a dummy for the presence of more than one dominant bidder in the corresponding auction. Table 4 reports the results.

Table 4 – OLS regression of dominant prices, P_{dom}
(310 observations).

Variables	P_{dom}
Intercept	1.8134 (1.97)
P_{resale}	0.0256 (0.93)
P_{small}	0.9725 (36.38)
Supply	1.92E-8 (0.38)
Demand small	-8.94E-10 (-0.05)
Other dominant	0.0046 (0.03)
R^2	0.9998

t-statistics in parenthesis, based on white heteroskedasticity-consistent standard errors.

Modeling prices of dominant bid vectors this way allows me to get an estimate of the mean (fitted) expected average price of a dominant bid vector in each auction as well as its standard deviation. From the point of view of a non-dominant bidder, a dominant bid vector would come from a random draw from this distribution of prices. However, in order to construct a vector of bids, based on the price drawn from this distribution, some additional assumptions regarding the total number of bids and their price-quantity levels are needed.

In the empirical implementation I describe in the next section, I have taken a simplified approach to construct the vector of dominant bids. I assume non-dominant bidders have good priors regarding the number of bids to be submitted by the dominant bidder, the quantity she demands at each bid, and the amplitude between the highest and lowest prices she bids. Taking these variables as given, I construct a vector of price-quantity bids such that the prices are equally distributed across the price range (amplitude) of the actual bid vector, having as average the price randomly drawn from the distribution of dominant bid vectors. Table 5 illustrates this procedure, using as example the dominant bid vector placed by bidder #1 in the Jan/06/98 T-bill auction examined above. An estimated P_{dom} of 919.4474 is considered.

Table 5 – Dominant bid vector estimation

Actual bid vector		Estimated bid vector	
Price	Quantity	Price	Quantity
919.9488	89,000	919.7803	89,000
919.6457	20,000	919.4474	20,000
919.2830	1,000,000	919.1145	1,000,000
Amplitude = 0.6658		Amplitude = 0.6658	
Avg. Price = 919.6258		Estimated Pdom = 919.4474	

The assumptions taken above may seem unrealistic. However, bidders exhaustively communicate with each other and know who the major market players are. Thus non-dominant bidders may indeed have good perceptions of when a dominant bid vector will be placed in a particular auction. It is also reasonable to assume that, based on prior events and on talks before the auctions, bidders may be able to identify which institutions are likely to enter the auction as dominant bidders and establish good estimates of the demand of these institutions.

The implementation of the suggested methodology that deals with the presence of dominant bidders is described below in six steps:

1. Separate bidders into two groups: dominant bidders and non-dominant bidders. A bidder is considered dominant if the total amount of his bids is greater or equal to 20% of the amount auctioned.
2. Fix bidder i among the total N non-dominant bidders in the auction.
3. From the sample of $(N - 1)$ bid vectors, draw a random sample of $(N - 1)$ bid vectors with replacement, giving equal probability of $1/(N - 1)$ to each bid vector in the original sample.
4. Based on moments found on a regression model for the dominant bid (price), draw a dominant bidvector for each dominant bidder j .
5. Construct the residual supply function generated by the $(N - 1)$ "resampled" non-dominant and the M dominant bid vectors.
6. Intersect with bidder i 's bid vector to find the market clearing price.

Repeat B times for each (non-dominant) bidder and for all (non-dominant) bidders in the data set.

3. Implementing the methodology with Brazilian Treasury Data

In this section I apply the methodology described in the previous section to data from auctions of Brazilian treasury 3-month fixed-rate securities held in January and February of 1998. A total of 6 auctions are examined, 4 of which included participation by dominant bidders (i.e., institutions that demanded more than 20% of the amount being auctioned).

Auctions of Brazilian treasury securities are under the responsibility of the National Treasury Secretariat (NTS), within the Ministry of Finance. The NTS sells fixed-rate securities through discriminatory (pay-as-bid) auctions open to all institutions listed in the “Sistema de Liquidação e de Custódia” (SELIC), a clearance and settlement system managed by the Central Bank which among other things registers the transactions with government securities. Although approximately 560 institutions are listed in the SELIC, a much lower number of institutions usually participate in the auctions (see table 6).

The amount of each security to be sold and its maturity are typically announced by the NTS two working days before the auction. On the auction date, each bidder is allowed to bid for the whole amount auctioned and to submit up to 5 different sealed bids before 1:30 p.m. After observing the bids the NTS decides whether to sell its securities entirely or to cancel the sale partially or totally. Auction results are released on the same day after 3:00 p.m. reporting the total amount sold of each security, the minimum and the weighted average price of the winning bids. Finally, in the first working day after the NTS auctions, the newly issued debt is allocated to the winning bidders while their reserve accounts are debited by the Central Bank.

Table 6 – Summary Statistics of the 6 auctions examined

Auction	Date	Supply (# of T-bills)	Demand	Non-dominant bidders	Dominant bidders	Number of bids
1	01/06/98	2,500,000	5,977,500	51	1	171
2	01/20/98	3,000,000	5,437,000	43	3	151
3	01/27/98	3,200,000	6,202,750	49	1	153
4	01/30/98	5,000,000	8,477,500	71	1	220
5	02/03/98	3,000,000	7,755,250	77	0	254
6	02/10/98	3,500,000	7,439,150	73	0	204

For auctions 5 and 6, which do not contain dominant bidders, the methodology used to reconstruct the marginal valuations of bidders coincides with that suggested by Hortaçsu. In the first four

auctions, however, dominant and non-dominant bidders are treated separately. While the sample of non-dominant bid vectors are used in steps 2 and 3 described above, dominant bid vectors are drawn in step 4 from a normal distribution with mean and standard deviation given by the regression model shown previously.

I run the methodology proposed in the previous section for each auction separately and obtain improved results. Making the bid vector of dominant bidders a random variable, instead of including it in the sample from which the bootstrap method is applied, provides much more reasonable estimates of marginal valuations of non-dominant bidders. The number of cases which such valuations take unrealistically high levels (sometimes going to infinity) are drastically reduced. In order to draw a comparison between Hortaçsu's and the proposed methodology, table 7 presents the marginal valuations corresponding to each bid submitted by bidder #2 in auction #1 according to the two estimation methods. In both cases the distribution of residual supply functions is constructed by keeping bidder #2 out of the sample and conducting 10,000 simulations (i.e., $B = 10,000$)

Table 7 – Estimated Marginal Valuations implicit in Bidder #2's bids
Hortaçsu's vs. Modified Method

Price bid	Valuation	Valuation
	Hortaçsu	Modified Method
919.2820	919.2820	919.2820
919.4335	919.4927	919.9412
919.5851	921.8094	920.2311
919.7366	2434.585	921.0262
919.8882	(infinite)	922.7086

Observe that, contrary to Hortaçsu's methodology, the modified approach allows the estimation of bidder #2's unobservable valuations for each of her 5 bids. Similar results are obtained if we examine the price bid by other bidders and their corresponding estimates of marginal valuations.

4. Testing bidder heterogeneity

The estimates of bidder's marginal valuations obtained through methods such as the one described above open a wide array of issues that can be investigated and that are of interest to auction researchers and policy makers. The ongoing discussion regarding the choice between discriminatory or uniform price auctions, that dates back to at least Friedman (1960), for example, has been addressed by

Hortaçsu (2000) and Heller and Lengwiler (1998). Based on structural models these authors test which mechanism would have provided higher revenues to the Turkish and Swiss treasuries, respectively.

Another important question is the possible existence of systematic differences in valuations across bidders' types. Most empirical studies of government securities conduct tests of auction literature predictions assuming a common values setting. Bidders are treated as if they all had the same value for the security, typically reflected in resale or when-issued markets. A growing set of evidence in the empirical literature, however, suggests that private components of bidders' valuations may play an important role on their strategies.³

The imposition of liquid asset reserve requirements by the Central Bank and the lack of liquidity in the secondary market are some of the examples that the auction literature provides to justify the effects of private valuation components in some countries. To the extent that these or other components are likely to have distinct impacts across bidders' categories, it becomes worthwhile to examine whether bidders with different characteristics apply different bidding strategies.

In order to carry out such an analysis, the investigator needs access to data at the individual bidding level. Few studies have had the required data, none of them was able to both classify bidders in categories and apply a structural methodology that allows the estimation of bidder's unobservable valuations.

In the analysis conducted below, I classify bidders according to their institutional category (commercial banks (Cbank), investment banks (Ibank), or brokers) and their nationality (national (Nbank) or foreign banks (Fbank)). The estimates of bidders' marginal valuations obtained in the previous section allows me to compute the elasticity of demand of each vector of bids presented by individual bidders. I then investigate whether elasticity levels differ across bidders' types.

Although the Brazilian Central Bank does not impose liquid asset reserve requirements, several factors, most of which related to the lack of liquidity in the secondary market for Brazilian T-bills, may provoke differences in the elasticity of demand across the categories of bidders described above. Bidders with limited flexibility with regards to the allocation of resources (i.e., institutions with fewer investment opportunities) are likely to present more inelastic demand compared to other bidders. This is because should they fail to win the auction, they would be more severely hurt by the high spreads in the secondary market. According to this argument commercial banks, institutions that usually have a broad set of investment opportunities, should present higher elasticity of demand than brokers or investment banks.

³ See, for example, Hamao and Jegadeesh (1998), Hortaçsu (2000) and Silva (2001).

That some institutions place bids on behalf of their clients is another reason for possible differences in the elasticity of demand of auction participants. Pension funds, for example, are allowed to participate in Brazilian treasury auctions only through institutions that are listed in the SELIC system. To the extent that these clients may be mainly interested in buying securities to hold, they may present more inelastic demands than other institutions. Since there is a limit of 5 bids that can be submitted by each institution listed in the SELIC, smaller bidders are the ones that are most likely able to “rent” their bidding rights to clients. In this sense, brokers and foreign banks (typically the smallest types of bidders among the ones described) are expected to show more inelastic demands than larger bidders such as national banks.

The elasticity of demand of bidders may also be related to their borrowing capacity. Bidders that can easily borrow large sums of resources may place some bids at low prices even knowing that they have limited probability of winning the auction. Should they end up winning the auction at these advantageous price levels they are able to raise the amount of money necessary to comply with their offer. Given that national, especially commercial, banks are the kind of institutions that best fit in this description, once again one should expect this type of bidders to present higher elasticity of demand than other types of auction participants.

Table 8 shows the number of bid vectors submitted by each category of bidder in the 6 auctions under examination. Bid vectors that could not be clearly classified in any category are denominated NC (not classified).

Table 8 – Number of bid vectors per bidder category*

Category	# of bid vectors	Category	# of bid vectors	Category	# of bid vectors
CBANK	133	NBANK	107	Ncbank	74
IBANK	43	FBANK	69	Fcbank	59
BROKER	64	NC	42	Nibank	33
NC	42			Fibank	10
				Nbroker	61
				Fbroker	3
				NC	42
Total	282	Total	218	Total	282

*Includes price-quantity vectors submitted in the auctions described in table 6

Observe that commercial banks presented most bid vectors 133, with active participation of both national and foreign Cbank (74 and 59 bid vectors, respectively). Also note the relevance of national brokers (as opposed to foreign brokers) in the market for Brazilian T-bills. Out of the 64 bid vectors credited to brokers, 61 were submitted by Brazilian institutions.

The measure of elasticity that I adopt in this study represents the elasticity of demand taking into consideration the first (highest) and last (lowest) value-quantity pairs of each bid vector. This variable, that I denote e_{mimax} , is then regressed against dummies for bidders' categories and control dummies for the auction environment. Due to the existence of some outliers in the distribution of e_{mimax} , I exclude from the sample of observations those elasticities located in the upper or lower 5% of its distribution.

Table 9 reports the regression results. Note that in line with the argument that national commercial banks have higher elasticity of demand than other types of bidders, its coefficient in the first regression is negative and significant at the 5% level. That bidders are heterogeneous with respect to their elasticity of demand constitutes additional evidence that suggests caution in the interpretation of results that rely on the common values assumption in auctions of treasury securities.

The second and third regressions provide additional evidence that commercial and national banks, respectively, present higher elasticity of demand than those of other categories of bidders. The coefficients of Cbank and Nbank are negative, however their significance levels decrease to approximately 7% and 10%.

Table 9 – Elasticity of demand OLS regressions
(214 observations)

Variables	e_{mimax}	e_{mimax}	e_{mimax}
Intercept	-858.04** (-11.48)	-880.79** (-12.75)	-898.49** (-12.04)
auction1	210.25* (2.35)	215.39* (2.46)	218.72* (2.41)
auction2	170.37 (1.65)	171.94 (1.65)	171.24 (1.71)
auction3	65.68 (0.59)	72.55 (0.66)	77.92 (0.69)
auction4	88.22 (1.01)	91.49 (1.05)	92.07 (1.04)
auction5	-156.03 (-1.67)	-157.12 (-1.69)	-153.00 (-1.63)
Ncbank	-145.77* (-2.06)		
Fcbank	-93.13 (-1.31)		
Nibank	-96.17 (-0.96)		
Fibank	70.87 (0.65)		
Cbank		-101.68 (-1.82)	
Nbank			-92.26 (-1.66)
R^2	0.1070	0.0979	0.0962

t-statistics in parenthesis, based on white heteroskedasticity-consistent standard errors.

* significant at 5% level

**significant at 1% level

Although bidder heterogeneity is the focus of the investigation conducted above, Table 9 also suggests that bidders' elasticity of demand vary across auctions. More specifically, bidders seem to have shown lower elasticity of demand in auction 1 and 2. It is reasonable to assume that these elasticities should be affected by other auctions of similar securities held simultaneously. In the days that the 6 auctions of 3-month Brazilian T-bills under investigation were held, the Brazilian treasury also auctioned T-bills with different maturities. A way to evaluate bidders' preferences across the different maturities of securities being auctioned is through the ratio between demand and supply (known as coverage ratio) in

their respective auctions. In this sense I observe that auctions of 3-month securities presented higher coverage ratios exactly in auctions 1 and 2. Another possible explanation to the lower elasticity of demand in auction 2 is the fact that the treasury refused to sell any securities in the auction held in the previous week. Therefore this demand shortfall may have affected the elasticity of demand of investors.

5. Concluding remarks

Recovering bidders' unobservable valuations in auctions of multiple goods such as those of government securities conducted throughout the world has been a great challenge to auction literature. Although Hortaçsu (2000) provides a model of bidder behavior that is consistent with a strategic divisible good auction framework, I have shown that the empirical implementation originally suggested by the author present limited results in auctions with dominant bidders.

I propose a modified methodology that distinguishes between dominant and non-dominant bids and allows improved results. The low computational demand of such methodology represents a promising venue to a broader use of structural analysis in multi-unit auctions in the future.

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