Universidade de Brasília
Departamento de Economia

Série Textos para Discussão

Setting the Right Expectations:
A Note on Carl Walsh's Market Discipline Paper

Fábia A. de Carvalho
Banco Central do Brasil

Maurício S. Bugarin
Universidade de Brasília

Texto nº 286
Brasília, Abril de 2003

Department of Economics Working Paper 286
University of Brasilia, April 2003

UNIVERSIDADE DE BRASÍLIA
DEPARTAMENTO DE ECONOMIA

TEXTO PARA DISCUSSÃO Nº 286

Setting the Right Expectations: a Note on Carl Walsh's Market Discipline Paper

Fábia A. de Carvalho
Banco Central do Brasil
e
Maurício S. Bugarin
Universidade de Brasília

Brasília, 11 de abril de 2003

UNIVERSIDADE DE BRASÍLIA
DEPARTAMENTO DE ECONOMIA
Campus Universitário Darcy Ribeiro
Instituto Central de Ciências
Caixa Postal 04302, 70910-900 Brasília, DF, Brasil
Tel.: (55-61) 3072498, 2723548
Fax: (55-61) 3402311
E-mail: econ@unb.br
http://www.unb.br/ih/eco

Setting the right expectations: A note on Carl Walsh's market discipline paper

Fabia A. de Carvalho♣
Maurício S. Bugarin♦

Abstract°♦

This paper adjusts the way expectations are set in Walsh (2000) in order to better assess the impact of market rigidities and expectations on the optimal choices of inflation in a monetary game between society and central bankers. Within the corrected framework, optimal inflation is higher, suggesting that the time inconsistency phenomenon plays a more important role in explaining inflationary biases than originally interpreted in Walsh (2000). However, if society organizes itself towards shorter-tenure wage contracts, inflation will be lower. Moreover, a central banker committed to announced inflation targets will have here more opportunities to generate output growth above equilibrium rates. Finally, the contractual structure of the economy proposed by Walsh is shown to be generally unstable. Using a quadratic social welfare function indicates that society is better off by choosing longer-tenure wage contracts, moving away from shorter-tenure ones, at the cost of higher inflation.

♣ Branco Central do Brasil. E-mail: fabia.carvalho@bcb.gov.br.
♦ Universidade de Brasilia. E-mail: bugarin@unb.br. Please send correspondence to Mauricio Bugarin, Departamento de Economia, Universidade de Brasília, Caixa Postal 04302, 70910-900 Brasilia, DF, BRAZIL
°♦ We are especially grateful to Mirta Bugarin, Stephen de Castro and Ilan Goldfajn for their valuable comments and analysis. They are, nonetheless, exempted from any error or omission this paper may still present.

SETTING THE RIGHT EXPECTATIONS:
A NOTE ON CARL WALSH'S MARKET DISCIPLINE PAPER

ABSTRACT

This paper adjusts the way expectations are set in Walsh (2000) in order to better assess the impact of market rigidities and expectations on the optimal choices of inflation in a monetary game between society and central bankers. Within the corrected framework, optimal inflation is higher, suggesting that the time inconsistency phenomenon plays a more important role in explaining inflationary biases than originally interpreted in Walsh (2000). However, if society organizes itself towards shorter-tenure wage contracts, inflation will be lower. Moreover, a central banker committed to announced inflation targets will have here more opportunities to generate output growth above equilibrium rates. Finally, the contractual structure of the economy proposed by Walsh is shown to be generally unstable. Using a quadratic social welfare function indicates that society is better off by choosing longer-tenure wage contracts, moving away from shorter-tenure ones, at the cost of higher inflation.

1. INTRODUCTION

In the recent past, much has been done in the economic literature to try to assess the impact of market rigidities and the mix of backward- and forward-looking expectations in optimal choices of inflation, as a means to better connect theoretical models to economic reality. By analyzing the predictions of such models, economists have been able to identify mechanisms that could lead to higher discipline of central bankers.

After Kydland and Prescott (1977) and Calvo (1978) identified the time inconsistency problem in economics, Taylor (1979) formalized the idea that output growth can be affected by the dynamics inherent to nominal, rigid, and overlapping wage contracts based on forward-looking expectations. The incentive for central bankers to generate output growth through inflationary surprises was well depicted then. Several studies that followed these seminal papers attempted to identify channels for disciplining central bankers as a quest for price stability. Among those, special mention should be made to the very important papers by Barro and Gordon (1983b) (reputation), Rogoff (1985) (process of appointing the central banker), Walsh (1995a) (formal contracts and inflation targets), and more recently Walsh (2000) (market structure and uncertainties). In the latter, Walsh showed that: 1) by adding two period wage contracts to the economic structure of the game, the nature of the equilibrium may be altered due to the importance of the expectations to the output gap; 2) the standard prediction that a central banker that is strong on inflation leads to recession when his type is private information is not necessarily right.

In spite of the remarkable results achieved, Walsh's paper's construction of the way expectations are set need revising. In his model, the negotiation of one-period nominal wage contracts is based on information available at the period that precedes the contract's tenure. Nonetheless, when contracts that hold for two periods are considered, the expectations are set taking into consideration information that will only be available at the end of the first period of the contract's term.[1]

[1] One way to justify the calculations for the expected real wage for two-year contracts is to assume that at the end of the first period of the contract's life the real wage will be indexed to the past inflation and that at the end of the first period, the nominal wage for the second period can be renegotiated by using information only realized in that very period. However, this is a rather strong assumption and no explanations are given as to why nominal wages would be set in a different way depending on the wage contract's tenure.

This paper aims to adjust the way expectations are set in Walsh's paper and to test the accuracy of the original results. The assumptions implicit in the negotiation process of one-period contracts are extended to the negotiation of two-year contracts. The remaining assumptions are the ones used in Walsh (2002). The immediate consequence of this adjustment in expectations is the derivation of different functions to represent the output gap of the economy. In these new functions, inflation persistence endogenously affects the output gap, in spite of the assumption that expectations are rational and forward looking. This result is in line with the inheritance of rigidity from past negotiations of wage contracts lasting more than one period. In addition, optimal inflation choices are higher than those obtained in Walsh, suggesting that the time inconsistency in the revised model is more powerful to explain the inflationary bias. Another result that differs from Walsh's refers to output expansions by a conservative central banker. In the adjusted model, it is not true that under a pooling equilibrium, a central banker who is committed to his inflation announcements generates output growth below trend levels in the second period of his term of office.

Possibly the most important result obtained from the revised model is the prediction that a higher share of shorter-tenure contracts brings the optimal inflation down. As a result, if society aims at low inflation, Walsh's 3-sector model is not justifiable. However, in order to discuss society's optimum, one needs to define the social welfare function. In the last section of this paper, two types of social utility functions are introduced. Their values, according to the various possibilities of central bankers' decisions and output gap results, suggest that, in the first case, the three-sector model proposed by Walsh is stable under a game theoretic approach. Notwithstanding, this result should be looked at carefully, as it is a very particular case where symmetry plays an important role. In the second case, where inflationary surprises in any direction lead to utility losses, society is likely to migrate to longer-tenure contracts. Therefore, Walsh's basic assumptions on the way society is organized are unstable.

The paper is organized as follows. Section 2 presents the derivation of the macro model after correcting the way expectations are set. Section 3 presents the optimal choices of inflation and the predictions for the output gap under each type of equilibrium. Section 4 tests the stability of the contractual premises under a game theoretical approach. The last section concludes the paper.

2. THE MACROECONOMIC MODEL AFTER SETTING THE CORRECTED EXPECTATIONS

The basic framework follows the one described by Walsh (2000). The economy is divided in three sectors: A, B1 and B2. Sectors differ as to the length of their wage contracts and the timing of their wage negotiation. Sector A negotiates wage contracts that last for one period, whereas sectors B1 and B2 negotiate contracts that last for two periods, staggering in the first period. The production function of each sector's representative firm is a labor intensive Cobb-Douglas with decreasing returns of scale of the type $Y_t^i = \left(L_t^i\right)^{a_L}$, in which i=A, B1, or B2, and $0 < a_L < 1$.[2] By maximizing profits in each period t, the marginal productivity of the labor factor in each sector will equal its marginal cost, and in logs, that yields:

$$\eta_t^i = \frac{1}{1-a_L}(p_t - w_t^i) + \frac{a_L}{1-a_L} \tag{1}$$

in which $\eta_t^i = \log L_t^i$, $p_t = \log P_t$, and $w_t^i = \log W_t^i$, with P_t and W_i^t representing the price level and nominal wage at time t, respectively.[3]

In each sector, wage setters negotiate nominal wages so as to keep the expected real wage unchanged throughout time. In turn, workers supply any amount of labor demanded by firms. In sector A, contracts are negotiated for one period. At the end of period t–1, wage setters negotiate the nominal wage W_t^A for period t, thus generating expectations as to inflation in period t. The targeted real wage W^* in logs will thus be $w^* = w_t^A - E_{t-1}p_t$.[4] The output gap representative of sector A will then be given by:

$$y_t^A = \frac{a_L}{1-a_L}\left(\pi_t - E_{t-1}\pi_t\right) \tag{2}$$

---

[2] Note that, as the production function of each firm shows decreasing returns of scale, or $nY_t^i = n\left(L_t^i\right)^{a_L} > \left(nL_t^i\right)^{a_L} = n^{a_L}\left(L_t^i\right)^{a_L}$, the economy as a whole cannot be represented by a standard aggregate production function. Later, the function representative of the economy will be an average of the ones representative of each sector.

[3] Note that the price is invariant to the sector. In addition, as the technological factor is the same for the three sectors, one needs to assume that the goods produced are also invariant to the sector where they are produced. As a result, it is necessary to assume input immobility within the economy and that within each sector only one type of contract can be negotiated. These assumptions are not explicit in Walsh (2000).

[4] From $W^* = \frac{W_t^A}{E_{t-1}P_t}$. Detailed calculation for the output gap can be found in the appendix.

where π_t is inflation in period *t* and $E_{t-1}\pi_t$ represents inflation expectations for period *t* based on information available at the end of period *t*–1.The details of the output gap derivation for sector A can be found in Appendix A, as they are essentially the same as those shown in Walsh (2000).

There is no reason to assume that each sector will have a different *rationale* for setting expectations. Adopting the same methodology used for sector A, wage setters in sector B1 will negotiate their contracts, which will be effective at *t* and *t*+1, based on the information available at the end of *t*–1, which is when they sign their contracts. Wage setters in B2, in turn, utilize the information available at the end of *t*–2 to negotiate their contracts (effective at *t*–1 and *t*). Therefore, the expected real wages in B1 will be $\frac{W_t^{B1}}{E_{t-1}P_t}$ at *t* and $\frac{W_{t+1}^{B1}}{E_{t-1}P_{t+1}}$ at *t*+1. But $W_t^{B1} = W_{t+1}^{B1}$, as the nominal wage will be fixed during the lifetime of the contract. Accounting for the fact that the average expected real wage at the time of the negotiation will be given by the geometric average of the expected real wages in each period, discounted intertemporally, the real expected wage in sector B1 will equal $W^* = \frac{W_t^{B1}}{E_{t-1}P_t} \times \left(\frac{W_t^{B1}}{E_{t-1}P_{t+1}}\right)^{\rho}$, in which ρ is the intertemporal discount factor, with $0 < \rho < 1$. Assuming that $W^* = 1$, since wage setters' goal is to keep the real purchasing power constant over the lifetime of the contract, in logs that will yield:

$$w_t^{B1} - E_{t-1}p_t + \rho\left(w_t^{B1} - E_{t-1}p_{t+1}\right) = 0 \qquad (3)$$

Equation (3) differs substantially from the one obtained in Walsh (2000): $w_t^{B1} - p_t + \rho\left(w_t^{B1} - E_t\, p_{t+1}\right) = 0$. Walsh's result implies that in the end of the first period of the contract, wages will be indexed to contemporaneous inflation. In addition, in spite of negotiating their contracts at the end of period *t*–1, wage setters will have a chance to update their expectations for period *t*+1 based on the information on inflation at period *t*. In fact, it is as if they were able to renegotiate their nominal contracts for the second period halfway through the contract's life, which would turn them almost into a one-period contract (as the ones in sector A). The results of the model using that simple equation are thus seriously jeopardized.

From (3), the actual real wage in sector B1 will be given by[5]:

$$w_t^{B1} - p_t = E_{t-1}\pi_t - \pi_t + \frac{\rho}{1+\rho} E_{t-1}\pi_{t+1} \tag{4}$$

Replacing (4) into the Cobb-Douglas production function representative of sector B1 yields the (log) level of the output in sector B1:

$$\log Y_t^{B1} = a_L \eta_t = a\frac{a_L}{1-a_L} + \frac{a_L}{1-a_L}(\pi_t - E_{t-1}\pi_t) - \frac{a_L}{1-a_L}\frac{\rho}{1+\rho}E_{t-1}\pi_{t+1} \tag{5}$$

Equilibrium output level $\tilde{Y}_t^{B1}$ is defined as that in which there are no price surprises or shocks in the economy. When there are no price surprises, $\pi_t = E_{t-1}\pi_t$. Assuming that in equilibrium inflation is constant and equal to λ, and more particularly $\lambda = 0$, then $\log \tilde{Y}_t^{B1} = a\frac{a_L}{1-a_L}$. This assumption is also necessary to obtain the results in Walsh (2000), although it is not explicitly stated. It is consistent, though, with the standard assumption that unanticipated inflation does not affect equilibrium output. As a result, output gap in sector B1 at *t* will be given by:

$$y_t^{B1} = \frac{a_L}{1-a_L}\left[(\pi_t - E_{t-1}\pi_t) - \frac{\rho}{1+\rho}E_{t-1}\pi_{t+1}\right] \tag{6}$$

which is significantly different from the equation found in Walsh (2000): $y_t^{B1} = -\frac{a_L}{1-a_L}\left(\frac{\rho E_t \pi_{t+1}}{1+\rho}\right)$.

By applying the way expectations are set in sector B1 to sector B2, the real expected wages in sector B2 at *t*–1 and *t* will be given by $\frac{W_{t-1}^{B2}}{E_{t-2}P_{t-1}}$ and $\frac{W_{t-1}^{B2}}{E_{t-2}P_t}$, respectively. This means that upon negotiation of contracts that will be effective at *t*–1

---

[5] See appendix A for a derivation of this equation.

and t, wage setters have not yet observed the actual inflation in those periods. As a result, the only information that is available to them is inflation at time t–2. Assuming that wage contracts are negotiated so as to maintain the expected real purchasing power constant during the lifetime of the contract, the average discounted real wage will be given by $W^{B2*} = \frac{W_{t-1}^{B2}}{E_{t-2}P_{t-1}} \times \left(\frac{W_{t-1}^{B2}}{E_{t-2}P_t}\right)^{\rho} = 1$. Rearranging terms yields, in logs:

$$w_{t-1}^{B2} = \frac{E_{t-2}p_{t-1} + \rho E_{t-2}p_t}{1+\rho} \tag{7}$$

The actual real wage in sector B2 at period t will thus be[6]:

$$w_{t-1}^{B2} - p_t = \left(E_{t-2}\pi_{t-1} - \pi_{t-1}\right) - \pi_t + \frac{\rho}{1+\rho}E_{t-2}\pi_t \tag{8}$$

First-order conditions in logs for the optimization problem of sector B2 result in $a_L + (a_L - 1)\eta_t^{B2} = w_{t-1}^{B2} - p_t$[7]. Plugging in into (8) yields the employment level at sector B2 at period t:

$$\eta_t^{B2} = \frac{a}{1-a_L} + \frac{1}{1-a_L}\left(\pi_{t-1} - E_{t-2}\pi_{t-1}\right) + \frac{1}{1-a_L}\pi_t - \frac{1}{1-a_L}\frac{\rho}{1+\rho}E_{t-2}\pi_t \tag{9}$$

Plugging in (9) into sector B2's Cobb-Douglas production function ($\log Y_t^{B2} = a_L\eta_t^{B2}$) yields the actual output level (in logs) in sector B2 at time t:

$$\log Y_t^{B2} = a\frac{a_L}{1-a_L} + \frac{a_L}{1-a_L}\left(\pi_{t-1} - E_{t-2}\pi_{t-1}\right) + \frac{a_L}{1-a_L}\pi_t - \frac{a_L}{1-a_L}\frac{\rho}{1+\rho}E_{t-2}\pi_t \tag{10}$$

When there are no price surprises, $\pi_{t-1} = E_{t-2}\pi_{t-1}$. Assuming that in equilibrium inflation is constant and equal to λ, and more particularly $\lambda = 0$, the equilibrium output

[6] See Appendix A for a detailed derivation.

[7] From $a_L\left(L_t^{B2}\right)^{a_L-1} = \frac{W_{t-1}^{B2}}{P_t}$.

in logs will then be given by $\log \tilde{Y}_t^{B2} = a \dfrac{a_L}{1-a_L}$. Thus, the output gap in sector B2 at time t will be given by:

$$y_t^{B2} = \frac{a_L}{1-a_L}\left[\left(\pi_{t-1} - E_{t-2}\pi_{t-1}\right) + \pi_t - \frac{\rho}{1+\rho} E_{t-2}\pi_t\right] \tag{11}$$

Note that equation (11) differs significantly from the one obtained in Walsh (2000): $y_t^{B2} = \dfrac{a_L}{1-a_L}\left(\pi_t - \dfrac{\rho}{1+\rho} E_{t-1}\pi_t\right)$.

In the presence of decreasing returns of scale, the economy cannot be represented by a standard aggregate production function. In order to overcome this problem, Walsh (2000) defines the output gap representative of the economy as the geometric average of the output gaps of three sectors. The weights attributed to sectors A, B1 and B2 are γ, $\dfrac{1-\gamma}{2}$, and $\dfrac{1-\gamma}{2}$, respectively. Walsh interprets γ as the "fraction (…) of all firms and workers (that) set wages for one period at the start of each period" and $1-\gamma$ as the fraction of "all firms and workers (that) set nominal wages for two periods". However, this definition is inconsistent with the fact that employment is endogenous to the model, and thus the fraction of workers negotiating these contracts will be changing throughout time. In order to avoid this inconsistency, γ should represent here the fraction of sector A firms in the total number of firms in the economy. For that purpose, it is also necessary to assume that there are no entries or exits of firms within the sectors throughout time.

With n_i representing the number of firms in sector i, i=A, B1, or B2, the output gap in the economy at t will then be given by:

$$\frac{Y_t - \tilde{Y}_t}{\tilde{Y}_t} \cong \left[\left(\frac{Y_t^A - \tilde{Y}_t^A}{\tilde{Y}_t^A}\right)^{n_A} \times \left(\frac{Y_t^{B1} - \tilde{Y}_t^{B1}}{\tilde{Y}_t^{B1}}\right)^{n_{B1}} \times \left(\frac{Y_t^{B2} - \tilde{Y}_t^{B2}}{\tilde{Y}_t^{B2}}\right)^{n_{B2}}\right]^{\frac{1}{n_A+n_{B1}+n_{B2}}}$$

$$\cong \left[\left(\frac{Y_t^A - \tilde{Y}_t^A}{\tilde{Y}_t^A}\right)^{\gamma} \times \left(\frac{Y_t^{B1} - \tilde{Y}_t^{B1}}{\tilde{Y}_t^{B1}}\right)^{\frac{1-\gamma}{2}} \times \left(\frac{Y_t^{B2} - \tilde{Y}_t^{B2}}{\tilde{Y}_t^{B2}}\right)^{\frac{1-\gamma}{2}}\right] \tag{12}$$

in logs, equation (12) becomes:

$$y_t \cong \gamma y_t^A + \left(\frac{1-\gamma}{2}\right) y_t^{B1} + \left(\frac{1-\gamma}{2}\right) y_t^{B2} \qquad (13)$$

Plugging in (2), (6) and (11) into (13) yields the output gap of the economy at time *t*:

$$y_t = \bar{a}\left[\pi_t + \frac{1-\gamma}{2}\left(\pi_{t-1} - E_{t-2}\pi_{t-1}\right) - \frac{1+\gamma}{2} E_{t-1}\pi_t - \frac{1-\gamma}{2}\bar{\rho}\left(E_{t-1}\pi_{t+1} - E_{t-2}\pi_t\right)\right] \qquad (14)$$

where $\bar{a} = \frac{a_L}{1-a_L}$ and $\bar{\rho} = \frac{\rho}{1+\rho}$. This equation is significantly different from that obtained in Walsh, mainly due to the persistence of inflation (if not perfectly anticipated) affecting the output gap at time *t*. This result is consistent with the inheritance of rigidity from past wage negotiations.

Similarly to the equation for the output gap at *t*, output gap at period *t*+1 will be given by:

$$y_{t+1} = \bar{a}\left[\pi_{t+1} + \frac{1-\gamma}{2}\left(\pi_t - E_{t-1}\pi_t\right) - \frac{1+\gamma}{2} E_t\pi_{t+1} - \frac{1-\gamma}{2}\bar{\rho}\left(E_t\pi_{t+2} - E_{t-1}\pi_{t+1}\right)\right] \qquad (15)$$

Note that distinctly from Walsh, the parameter $\bar{a}$, which impacts multiplicatively the output deviation from its equilibrium level in both periods, does not depend on the share of sector A, γ, in the total number of firms.

3. DERIVING NEW EQULIBRIUM RESULTS

The strategic interaction between society and the central banker is the same as in Walsh (2000). A summary of how the game evolves and a thorough derivation of the equilibria are provided in Appendix B. A comparative chart of the inflation and output results derived under Walsh's model and under the one in which expectations are set correctly is shown in Tables 1 to 3 below[8].

The appendix describes the game in detail. Its general features are the following. Every other period a new central banker is appointed and announces inflation targets for the two periods (*t* and *t*+1) of his tenure. The central banker has total control over inflation and could be of two different types; the conservative S-type central banker

[8] The focus of the adjusted model is on the separating and pooling equilibria. Walsh (2000) also derives the results under the mixed equilibrium.

always delivers on its announced inflation targets whereas the W-type is opportunistic and may not implement its announced target. The central banker's type is his private information and society only knows the ex-ante probability q that the central banker is of type S. After the announcement of the inflation policy, society forms expectations on the central banker's type and all one-period and half of the two-period wage contracts are negotiated. Next, the central banker chooses and implements the first period inflation rate, society updates its beliefs about the central banker's type and negotiates the new one-period and the other half of the (staggered) two-period contracts. At the beginning of the second period a new central banker is appointed and the game repeats itself. The extensive form of the two-period interaction just described can be found in the appendix.

The superscript a which appears in the tables below stands for "announced". The signs of inflation outcomes do not vary from one model to the other, except for the inflation announced for period $t+1$ in the separating equilibrium. By correcting the expectations setting, optimal inflation announced and fulfilled by a conservative central banker in the second period of a separating equilibrium will usually be positive (it will be null only if all contracts in the economy last for one period, which is not the case analyzed in Walsh). This result seems to be more intuitive than that found in Walsh (2000) as, after society observes the type of the central banker, revealed at the end of the first period, only those renewing their contracts will be able to lower their expectations. In the adjusted model, two-period contracts signed at the beginning of the first period cannot be renegotiated in the middle of their lifetime. Thus, the best that wage setters can do is to attribute some probability of the central banker being of one type or another at period $t+1$. Therefore, inflation under an opportunistic central banker will still exert pressure on expectations, and if the central banker chooses too low an inflation, his choice may lead to unnecessary recession in the economy.

Table 1

Results of the Models: Analyzing the Sign of Inflation Outcomes

Equilibrium	Walsh (2000)	Revised expectations setting
Separating Equilibrium	$\pi_{t+1}^{W} = \frac{\alpha}{\beta} > 0$	$\pi_{t+1}^{W} = \frac{\bar{a}}{\beta} > 0$
	$\frac{\partial \pi_{t+1}^{W}}{\partial \gamma} > 0$	$\frac{\partial \pi_{t+1}^{W}}{\partial \gamma} = 0$
	$\pi_{t}^{W} = \frac{\alpha}{\beta} = \pi_{t+1}^{W} > 0$	$\pi_{t}^{W} = \left(1 + \left(\frac{1-\gamma}{2}\right)\rho\right)\pi_{t+1}^{W} > \pi_{t+1}^{W}$
	$\frac{\partial \pi_{t}^{W}}{\partial \gamma} > 0$	$\frac{\partial \pi_{t}^{W}}{\partial \gamma} < 0$
	$\pi_{t}^{a} = \frac{\alpha}{\beta}\left[1 - (1-k)q\right] < \pi_{t}^{W}$	$\pi_{t}^{a} = \left[1 + \frac{\rho(1-q)(1-\gamma) - q(1+\gamma)}{2}\right]\pi_{t+1}^{W} > 0$
	$\frac{\partial \pi_{t}^{a}}{\partial \gamma} < 0$ if $q < 1/2$ > 0 if $q > 1/2$	$\frac{\partial \pi_{t}^{a}}{\partial \gamma} < 0$
	$\pi_{t+1}^{a} = 0$	$\pi_{t+1}^{a} = \left(\frac{1}{2(1+\rho)}\right)(1-\gamma)(1-q+\rho(1+q))\pi^{d} > 0$ (if $\gamma = 1$ then $\pi_{t+1}^{a} = 0$)
	$\frac{\partial \pi_{t+1}^{a}}{\partial \gamma} = 0$	$\frac{\partial \pi_{t+1}^{a}}{\partial \gamma} < 0$
Pooling Equilibrium	$\pi_{t} = k\frac{\alpha}{\beta} < \pi_{t}^{W}\ (separating)$	$\pi_{t}^{a} = \left(\frac{1-\gamma}{2}\right)\left(\frac{\bar{a}}{\beta}\right) > 0$ (if $\gamma = 1$ then $\pi_{t}^{a} = 0$)
	$\frac{\partial \pi_{t}^{a}}{\partial \gamma} < 0$	$\frac{\partial \pi_{t}^{a}}{\partial \gamma} < 0$
	$\pi_{t+1}^{a} = (1-q)\frac{\alpha}{\beta} < \pi_{t+1}^{W}$	$\pi_{t+1}^{a} = \frac{1}{1+\rho}(1-q+\rho(1-\gamma q))\pi_{t+1}^{W} > 0$
	$\frac{\partial \pi_{t+1}^{a}}{\partial \gamma} > 0$	$\frac{\partial \pi_{t+1}^{a}}{\partial \gamma} < 0$

This rationale also applies to the pooling equilibrium. The conservative central banker will usually have to inflate at a positive rate in both periods, bearing the burden of the uncertainty as to his type. Notwithstanding, optimal inflation announced for the two periods will be lower than the optimal discretionary rates.

In addition, distinctly from Walsh, discretionary inflation in the separating equilibrium varies depending on the time period. This seems to be more intuitive, as actual unanticipated inflation at time *t* affects the output gap not only at time *t*, but also at *t*+1. Therefore, there is additional incentive to raise inflation at *t*.

If the common sense that society benefits from low inflation applies, the ideal composition of contracts under the adjusted model is that in which $\gamma = 1$, as for any time period and any equilibrium or type of central banker, the sensitivity of inflation to shorter tenure contracts is decreasing. In Walsh (2000), the direction of γ's influence in inflation varies according to time periods, equilibriums and types of central banker, which does not allow for a conclusion of which contractual composition is the best. However, it is not straightforward to conclude that the 3-sector model is unstable under the adjusted expectations setting, as the social utility function has not been defined. In the following section, two alternative functions will be described to allow for testing the hypothesis that lower γ increases social welfare.

Table 2 shows the comparison of the value of optimal choices under Walsh's model and the one in which expectations are set correctly. In the model with corrected expectations setting, all inflation results are higher than those obtained in Walsh when there are different tenures of wage contracts in the economy. Therefore, time inconsistency is more powerful to explain the inflationary bias that arises with the presence of more than one-period rigidities in the economy. In other words, Walsh (2000)'s results underestimate the role time inconsistency plays in generating high inflation.

Table 2

Compared Results

a = results obtained in Walsh (2000)

b = results obtained with the adjusted informational structure

Equilibrium	**a–b**	**If** $\gamma \neq 1$	**If** $\gamma = 1$
π_t^W (separating)	$-\frac{a_L}{2}\left[\frac{(1-\gamma)(1+\rho)}{\beta(1-a_L)}\right]$	< 0	$= 0$
π_{t+1}^W	$-\frac{a_L}{2}\left[\frac{1-\gamma}{\beta(1-a_L)}\right]$	< 0	$= 0$
π_t^a (separating)	$-\frac{a_L}{2}\left[\frac{(1-\gamma)\rho^2+(2-q)\rho+(1-q)}{(1+\rho)\beta(1-a_L)}\right](1-\gamma)$	< 0	$= 0$
π_{t+1}^a (separating)	$-\frac{a_L}{2}\left[\frac{1-q+\rho(1+q)}{(1+\rho)\beta(1-a_L)}\right](1-\gamma)$	< 0	$= 0$
π_t^a (pooling)	$-\frac{a_L}{2}\left[\frac{(1-\gamma)\rho}{(1+\rho)\beta(1-a_L)}\right]$	< 0	$= 0$
π_{t+1}^a (pooling)	$-\frac{a_L}{2}\left[\frac{1-q+\rho(1+q)}{(1+\rho)\beta(1-a_L)}\right](1-\gamma)$	< 0	$= 0$

Table 3 shows the signs of the output gap under the model with corrected expectations setting. The algebraic expressions are derived in Appendix C. The signs were obtained through analysis of an *n*-grid formed by possible discrete values of the variables, utilizing the Matlab software. The most striking difference from Walsh's results lies on the fact that a conservative central banker under a pooling equilibrium will achieve a positive output gap in the second period of his term, whereas in Walsh the output will be below trend levels.

Table 3

Output Gap under Corrected Expectations Setting

Equilibrium	Type of Central Banker	Time	Sign of Output Gap
Separating Equilibrium	S	t	>0, if $\gamma \le 0.5$ =0, if $\gamma = 1$ and $q = 0$ or1 non-defined if $\gamma > 0.5$
	W	t	>0, if $q \ne 0$ and $\gamma \ne 1$
	S	t+1	>0, if $\gamma \le 0.1$ and $q \ge 0.9$ =0, if $\gamma = 1$ non-defined otherwise
	W	t+1	>0, if $\gamma \ge 0.4$ =0, if $\gamma = 1$ non-defined otherwise
Pooling Equilibrium	S at t–1	t	>0, if $\gamma < 0.3$ and $\gamma \ne 1$
	W at t–1	t	>0, if $q > 0.5$ or $\gamma < 0.5$ and $\gamma \ne 1$
	S	t+1	>0
	W	t+1	>0, if $q \ne 0$ and $\gamma \ne 1$

4. TESTING STABILITY IN THE THREE-SECTOR MODEL

Should the share of firms negotiating shorter tenure wage contracts increase, optimal inflation choices under corrected expectations setting are lower. Notwithstanding, the socially optimal level of inflation depends on the assumptions for the social utility function. In this section, two alternative welfare functions are derived according to the premises of the original model in order to test the stability of the three-sector breakdown under a game theoretical approach.

4.1. Maximizing the Purchasing Power of Wages

As the explicit goal of wage setters in Walsh (2000) is to maintain (or even increase) the purchasing power of wages, the first social utility function considered will be that in which price surprises that reduce real wages generate welfare loss and, on the other hand, lower than expected inflation causes social welfare to rise through increases in the wages' purchasing power. Intertemporal social utility functions in each sector *i* at periods *t* and *t*+1 will be described by equation (16):

$$USoc^{i} = \left[E_{t-1}\pi_{t} - \pi_{t} + \rho\left(E_{t}\pi_{t+1} - \pi_{t+1}\right)\right] \tag{16}$$

In expected terms, the social utility in each sector will be a function of the probabilities assigned to each type of central banker, the superscripts S and W standing for the outcome of the games under each type of central banker:

$$E\left[USoc^{i}\right] = \left\{qUSoc^{i}(S) + (1-q)USoc^{i}(W)\right\} \tag{17}$$

In the separating equilibrium, replacing the variables in (17) by the corresponding game results according to each type of central banker and sector yields:

<u>Type S central banker:</u>

$$USoc^{A}(S) = USoc^{B2}(S) = \left[(1-q)\left(\pi_{t}^{a} - \pi_{t}^{W}\right)\right] \tag{18}$$

$$USoc^{B1}(S) = \left[q(1-q)\left(\pi_{t}^{W} - \pi_{t}^{a}\right) + \rho(1-q)\left(\pi_{t+1}^{W} - \pi_{t+1}^{a}\right)\right] \tag{19}$$

<u>Type W central banker:</u>

$$USoc^{A}(W) = USoc^{B2}(W) = q\left[\pi_{t}^{a} - \pi_{t}^{W}\right] \tag{20}$$

$$USoc^{B1}(W) = q\left[\pi_{t}^{a} - \pi_{t}^{W} + \rho\left(\pi_{t+1}^{a} - \pi_{t+1}^{W}\right)\right] \tag{21}$$

Therefore, plugging in equations (18) to (21) into (17) yields the expected utility in each sector under the separating equilibrium:

$$E\left[USoc^{A}(Separating)\right] = E\left[USoc^{B1}(Separating)\right] = E\left[USoc^{B2}(Separating)\right] = 0 \tag{22}$$

Similarly, in the pooling equilibrium the expected utility function in each sector will be given by:

$$E\left[USoc^{A}(Pooling)\right] = E\left[USoc^{B1}(Pooling)\right] = E\left[USoc^{B2}(Pooling)\right] = 0 \tag{23}$$

It is straightforward from expressions (22) and (23) that the social welfare is the same under both types of equilibrium. As the conditions for one equilibrium to occur in detriment of the other depend directly on the weight of each sector in the economy $\left(\gamma, \frac{1-\gamma}{2}, \frac{1-\gamma}{2}\right)$, one concludes that the three-sector economic framework proposed by Walsh (2000) (after expectations are set correctly) is stable under a game theoretic approach, since there are no incentives for a sector to change its type of contracts to favor one equilibrium or another. However, it is not clear whether these results would be achieved if there was no symmetry in the weights attributed to sectors B1 and B2, or if society's utility was represented by a different type of function.

4.2. Minimizing Inflationary Surprises

Despite the fact that wage setters explicitly aim to maintain (or increase) the purchasing power of wages throughout time in Walsh (2000)'s model, this section introduces an alternative welfare function to account for the fact that rises in the purchasing power of wages lead to a reduction in employment levels, which are determined *ex-post* through the solution of the optimization problems of the firms. Therefore, surprises in any direction cause welfare loss. Similarly to the previous section, this section derives the expected utility in each sector according to the type of equilibrium and central banker, and tests the stability of the three-sector model under a game theoretic approach.

The intertemporal utility loss function in each sector at t and t+1 is described by equation (24), which is non-linear in the inflationary surprise:

$$LSoc^i = \left[(E_{t-1}\pi_t - \pi_t)^2 + \rho(E_t\pi_{t+1} - \pi_{t+1})^2\right] \tag{24}$$

The expected loss in each sector will be a function of the probabilities assigned to each type of central banker, as in equation (17), reproduced below:

$$E\left[LSoc^i\right] = \left\{qLSoc^i(S) + (1-q)LSoc^i(W)\right\} \tag{17}$$

Tedious calculations yield the expected loss in each sector in the separating equilibrium:[9]

$$E\left[LSoc^A(Separating)\right] = E\left[LSoc^{B2}(Separating)\right] = \frac{1}{4}a^2q^3(1-q)\frac{(1+\gamma+\rho(1-\gamma))^2}{\beta^2} > 0$$

[9] These calculations are available upon request.

$$E\left[LSoc^{B1}(Separating)\right]=\frac{1}{4\beta^2}a^2q\frac{(1-q)}{(1+\rho)^2}\begin{bmatrix}\rho^4\left(q^2\gamma^2-2q^2\gamma+q^2\right)\\+\rho^3\left(-6q^2\gamma+1+q^2\gamma^2+2\gamma+5q^2-2q+2q\gamma^2+\gamma^2\right)\\+\rho^2\left(4\gamma-4q^2\gamma^2+4q^2+4q^2\gamma+2+2\gamma^2\right)\\+\rho\left(\gamma^2+q^2\gamma^2-2q\gamma^2+5q^2+2q+2q^2\gamma+1+2\gamma\right)\\+2q^2\gamma+q^2\gamma^2+q^2\end{bmatrix}>0$$

Therefore, for $q\neq 0,1$,

$$\frac{\partial E\left[LSoc^{A}(Separating)\right]}{\partial\gamma},\ \frac{\partial E\left[LSoc^{B2}(Separating)\right]}{\partial\gamma},\ \frac{\partial E\left[LSoc^{B1}(Separating)\right]}{\partial\gamma}>0.$$

Thus, a higher presence of shorter tenure contracts increases the social loss in all sectors should an inflationary surprise occur, except when there is certainty on the type of central banker, in which case $\frac{\partial E\left[LSoc^{i}(Separating)\right]}{\partial\gamma}=0$.

The corresponding welfare functions for the pooling equilibrium are:

$$E\left[LSoc^{A,B2}(pooling)\right]=\frac{1}{4\beta^2}a^2q\rho(1-q)\frac{\left[q(1-\gamma)+\rho(1-q+\gamma(1+q))+1+\gamma\right]}{(1+\rho)^2}>0$$

$$E\left[LSoc^{B1}(Pooling)\right]=\frac{1}{4\beta^2}a^2q\frac{(1-q)}{(1+\rho)^2}\left(q-q\gamma+1+\rho+\gamma-\rho q+\rho\gamma+\rho q\gamma\right)^2$$

Therefore, for $q\neq 0,1$,

$$\frac{\partial E\left[LSoc^{A}(Pooling)\right]}{\partial\gamma},\ \frac{\partial E\left[LSoc^{B1}(Agregador)\right]}{\partial\gamma},\ \frac{\partial E\left[LSoc^{B2}(pooling)\right]}{\partial\gamma}>0$$

Hence, a higher presence of shorter-tenure contracts in the economy also reduces social welfare in the pooling equilibrium, unless there is certainty of the type of central banker, in which case $\frac{\partial E\left[LSoc^{i}(Pooling)\right]}{\partial\gamma}=0$.

The preferred equilibrium for each sector will be the one in which social loss is lower. The comparisons are in Table 4.

Table 4

Preferred Equilibrium

Sector	$E\left[LSoc^{i}(Separating)\right]-E\left[LSoc^{i}(Pooling)\right]$
A, B2	$\frac{1}{4\beta^{2}(1+\rho)^{2}}a^{2}q(1-q)\begin{bmatrix}\rho^{4}\left(q^{2}\gamma^{2}+q^{2}-2q^{2}\gamma\right)\\+\rho^{3}\left(-1-2q\gamma^{2}-q^{2}\gamma^{2}+3q^{2}+2q-2q^{2}\gamma-\gamma^{2}-2\gamma\right)\\+\rho^{2}\left(-2-2\gamma^{2}-4\gamma+8q^{2}-4q^{2}\gamma\right)\\+\rho\left(-1-2q-\gamma^{2}-q^{2}\gamma^{2}-2\gamma+6q^{2}\gamma+3q^{2}+2q\gamma^{2}\right)\\+q^{2}+q^{2}\gamma^{2}+2q^{2}\gamma\end{bmatrix}$
B1	$\frac{1}{4\beta^{2}}a^{2}q^{3}(1-q)(1+\gamma+\rho(1-\gamma))^{2}>0$ if $q\neq 1$ or $q\neq 0$.

For different combinations of the variables, different signs are found for the utility loss difference between equilibriums. However, proceeding with the numerical analysis of the expression by considering an *n*-grid of discrete possible values of the variables involved and utilizing Matlab, for any value of γ, $E\left[LSoc^{A,B2}(Separating)\right]-E\left[LSoc^{A,B2}(Pooling)\right]<0$ whenever $q\leq 0.40$. This implies that whenever the probability attributed to an opportunistic central banker is high, the separating equilibrium is preferred for sectors A and B2. For the cases where $q=1$ or $q=0$, which are the standard cases in the literature, sectors A and B2 will be indifferent between the two equilibriums. For $q>0{,}40$, both positive and negative results are achieved for $E\left[LSoc^{A,B2}(Separating)\right]-E\left[LSoc^{A,B2}(Pooling)\right]$. On the other hand, the pooling equilibrium is usually sector B1's first choice.

From the analysis of each of the three sectors, it is not straightforward to assure which equilibrium is socially preferred. However, for every sector, a higher share of firms negotiating one-period contracts reduces social welfare. As a result, the initial condition that the share of each sector in the economy is represented by $\gamma,\frac{1-\gamma}{2},\frac{1-\gamma}{2}$ is not stable under a game theoretical concept. This means that if society organizes itself towards two-period wage contracts, welfare will increase. From the sole viewpoint of inflation, as mentioned in the previous section, a higher share of firms negotiating one-period contracts has the potential to lower optimal inflation under any type of central banker. That implies that a utility function that incorporates the costs of inflation *per se* might generate different results.

5. CONCLUSION

Inflation expectations and price rigidities play a key role in central bankers' decisions, albeit acting in opposite directions. Whilst price rigidities create conditions for an opportunistic central banker to boost output growth above equilibrium levels, private agents attempt to anticipate these movements by updating their expectations. The standard result of such an interaction is high inflation with no output growth above trend levels. However, central bankers are not always opportunistic, and their types are not easily grasped *ex-ante*. The game that is then analyzed in this paper accounts for different types of central bankers, not known *ex-ante*, and different sets of price rigidities, *à la* Walsh. The way expectations are set in Walsh (2000), however, proved to hold severe problems. By correcting the setting of expectations in longer-tenure wage contracts and thus making it compatible with shorter-tenure contracts, the immediate result is a different function to represent the aggregate supply of the economy, and thus all the subsequent results from the strategic interaction between the central banker and society. In the aggregate supply function, inflation persistence (if not anticipated) affects the output gap. This result is in line with the inheritance of rigidities from past negotiations of longer-tenure contracts.

An important similarity of the results achieved after revising the expectations setting compared to those in Walsh (2000) refers to the prediction that, in a separating equilibrium, even a central banker who is highly committed to his announcements can generate output expansions above trend levels in the first period of his term of office. The amount of output growth, however, differs from one model to the other.

In spite of this first similarity, in Walsh (2000) that type of central banker is expected to drive output growth to levels below the trend in the second period of a pooling equilibrium, whereas under the revised expectations setting, the conservative central banker may generate output growth above trend levels. This result is particularly important, not only to a theoretical viewpoint, but to prove that by observing output expansions above trend levels one can not conclude that the central banker was lenient with inflation.

The equations that describe the optimal choices of each of the central bankers under the adjusted expectations setting exhibited a higher degree of complexity, especially the one that defines the requirements for an opportunistic central banker not to deviate from the game equilibrium under analysis. In spite of this complexity, multidimensional *n*-grid analysis in the Matlab derived the signs of inflation and output gaps.

Optimal inflation is higher under the adjusted model, which implies that the power of time inconsistency was underestimated in Walsh's model. Although different wage contracts' tenures add realism to the model, under the corrected expectations setting, a higher share of firms negotiating shorter tenure contracts will usually lower optimal inflation. Thus, if society is better off with lower inflation rates, longer-tenure contracts are not justifiable.

The inflation vs. output growth trade-off for society is not directly apparent in the game. A social utility function should be introduced, care being taken to preserve the importance attributed to the process of wage negotiation in the original model. The first social utility function that is analyzed in the present paper accounts for the fact that inflationary surprises that reduce the real purchasing power of wages cause utility losses, and those that raise the real purchasing power increase social welfare. Under this assumption, no economic sectors have incentives to change wage contracts' tenures, which characterizes stability under a game theory concept. Notwithstanding, this result should be accepted with care, as it directly relies on the perfect symmetry that was assumed for the share of firms negotiating longer-tenure contracts. The robustness of this result might be tested if the weight $\frac{1-\gamma}{2}$ assigned both to sectors B1 and B2 is endogenized to $\theta_j(1-\gamma)$ and $(1-\theta_j)(1-\gamma)$, respectively, where *j* stands for time.

Under an alternative social utility function, the stability of the adjusted model is tested for the case in which inflationary surprises in any direction cause welfare losses. In this case, for each one of the three economic sectors, a higher share of firms negotiating shorter-tenure contracts reduces social welfare. This result suggests that the original composition of different tenure wage contracts in the economy is not stable under a game theoretical approach, as the economy will bend towards longer-tenure contracts. Interestingly, society will accept higher inflation at the benefit of no surprises. In this case, stability and easy prediction of the future are more important assets in this economy. However, this utility function does not account for the costs of inflation *per se* upon society. In the real world, high inflation has proved to bear serious distortionary effects upon income distribution. In that sense, a possible extension of this paper would be to analyse the stability of the model under a social welfare function in which these costs are incorporated. Additionally, the possibility of reassignment of the incumbent after the end of his first term of office or uncertainty as to the actual lag of monetary policy may bring distinct and interesting predictions.

REFERENCES

Barro, R.J. & Gordon, D.B. (1983b). 'Rules, Discretion, and Reputation in a Model of Monetary Policy', *Journal of Monetary Economics*, **12**, 101-20

Bils, M. (1991). 'Indexation and Contract Length in Unionized US Manufacturing', in R.W. Eberts e E.L. Groshen (eds), 'Structure Changes in US Labor Markets', *M E Sharpe Press,* New York, NY e London.

Cukierman, A. & Liviatan, N. (1991). 'Optimal Accomodation by Strong Policymakers Under Incomplete Information', *Journal of Monetary Economics,* **27**, 99-127

Fischer, S. (1977a). 'Long Term Contracts, Rational Expectations, and the Optimal Money Supply Rule', *Journal of Political Economy*, **85**, 191-205.

Goodhart, C.A.E. & Huang, H. (1998). 'Time inconsistency in a model with lags, persistence, and overlapping wage contracts', *Oxford Economic Papers*, **50**, 378-96

Greenspan, A. (1994). 'Statement to Congress', *Federal Reserve Bulletin*, **July**, 594-609

Kydland, F.E. & Prescott, E.C. (1977). 'Rules Rather Than Discretion: The Inconsistency of Optimal Plans', *Journal of Political Economy*, **85**, 473-91

Maskin, E. & Tirole, J. (1988). 'A Theory of Dynamic Olipoly', *Econometrica*, 56, 549-99

Rogoff, K. (1985). 'The Optimal Commitment to an Intermediate Monetary Target', *Quarterly Journal of Economics*, **100**, 1056-70.

Romer, D. (1996). 'Advanced Macroeconomics', *The McGraw Hill Companies, Inc.*

Taylor, J.B. (1979). 'Staggered Wage Setting in a Macro Model', *American Economic Review*, **69**, 108-13

Walsh, C.E. (1995a). 'Optimal Contracts for Central Bankers', *American Economic Review*, **85**, 150-167.

Walsh, C.E. (1998). 'Monetary Theory and Policy', *The MIT Press*.

Walsh, C.E. (2000). 'Market Discipline and Monetary Policy', *Oxford Economic Papers*, **52**, 249-71

Appendix A. Derivation of equation (4) and (8)

Note that $w_t^{B1} - p_t = \dfrac{E_{t-1}p_t + \rho E_{t-1}p_{t+1}}{1+\rho} - p_t = \dfrac{E_{t-1}p_t - p_t + \rho(E_{t-1}p_{t+1} - p_t)}{1+\rho}$ but as

$E_{t-1}p_t - p_t = E_{t-1}p_t - p_{t-1} - (p_t - p_{t-1}) \cong E_{t-1}\pi_t - \pi_t$, and

$E_{t-1}p_{t+1} - p_t = E_{t-1}p_{t+1} - E_{t-1}p_t + E_{t-1}p_t - p_{t-1} - (p_t - p_{t-1}) \cong E_{t-1}\pi_{t+1} + E_{t-1}\pi_t - \pi_t$, it follows that

$$w_t^{B1} - p_t = \frac{E_{t-1}\pi_t - \pi_t + \rho(E_{t-1}\pi_{t+1} + E_{t-1}\pi_t - \pi_t)}{1+\rho} = \frac{(1+\rho)(E_{t-1}\pi_t - \pi_t) + \rho(E_{t-1}\pi_{t+1})}{1+\rho} =$$

$= E_{t-1}\pi_t - \pi_t + \dfrac{\rho}{1+\rho}E_{t-1}\pi_{t+1}$, which is equation (4).

Nota that $w_{t-1}^{B2} - p_t = \dfrac{E_{t-2}p_{t-1} + \rho E_{t-2}p_t - p_t - \rho p_t}{1+\rho} = \dfrac{E_{t-2}p_{t-1} - p_t + \rho(E_{t-2}p_t - p_t)}{1+\rho}$,

but as
$$\begin{aligned} E_{t-2}p_{t-1} - p_t &= E_{t-2}p_{t-1} - p_{t-1} - (p_t - p_{t-1}) = E_{t-2}p_{t-1} - p_{t-2} - (p_{t-1} - p_{t-2}) - (p_t - p_{t-1}) \\ &= E_{t-2}\pi_{t-1} - \pi_{t-1} - \pi_t \end{aligned}$$

and
$$\begin{aligned} E_{t-2}p_t - p_t &= E_{t-2}p_t - E_{t-2}p_{t-1} + E_{t-2}p_{t-1} - p_{t-2} + p_{t-2} - p_{t-1} + p_{t-1} - p_t \\ &= E_{t-2}\pi_t + E_{t-2}\pi_{t-1} - \pi_{t-1} - \pi_t = E_{t-2}\pi_t - \pi_t + E_{t-2}\pi_{t-1} - \pi_{t-1} \end{aligned},$$

the actual real wage can be expressed as

$$\begin{aligned} w_{t-1}^{B2} - p_t &= \frac{(E_{t-2}\pi_{t-1} - \pi_{t-1}) - \pi_t + \rho[(E_{t-2}\pi_{t-1} - \pi_{t-1}) + (E_{t-2}\pi_t - \pi_t)]}{1+\rho} \\ &= \frac{(1+\rho)(E_{t-2}\pi_{t-1} - \pi_{t-1}) - (1+\rho)\pi_t + \rho E_{t-2}\pi_t}{1+\rho} = (E_{t-2}\pi_{t-1} - \pi_{t-1}) - \pi_t + \frac{\rho}{1+\rho}E_{t-2}\pi_t \end{aligned}$$

Appendix B. The output gap in Sector A

In sector A, contracts are negotiated to last one period. At the end of period t–1, wage setters negotiate the nominal wage W_t^A for period t, thus forming expectations as to inflation in period t. The targeted real wage W^* in logs will thus be $w^* = w_t^A - E_{t-1}p_t$.[10] By normalizing $w^* = 0$, the actual real wage in period t, in logs, will be given by $w_t^A - p_t = E_{t-1}p_t - p_t$ [11]. Therefore, the employment level in A in logs will be:

$$\eta_t^A = \frac{1}{1-a_L}(p_t - E_{t-1}p_t) + \frac{a}{1-a_L}$$

where $a = \log a_L$ [12]. Replacing the equation for η_t^A in $\log Y_t^A = a_L \eta_t^A$ yields:

---

[10] From $W^* = \dfrac{W_t^A}{E_{t-1}P_t}$.

[11] With W^*=1, the nominal wage negotiated in logs will be $w_t^A = E_{t-1}p_t$.

[12] Equation (2) differs from the equation describing η_t^A presented in Walsh (2000) exactly by the amount $\dfrac{a}{1-a_L}$. Nonetheless, this term will be eliminated when the output gap for sector A is derived.

$$\log Y_t^A = \left(\frac{a_L}{1-a_L}\right)(p_t - E_{t-1}p_t + a) = \bar{a}(p_t - E_{t-1}p_t + a)$$

where $\log Y_t$ is the log of the output level in sector A at time *t*. The equilibrium output level ($\tilde{Y}_t$) will be that in which there are no price surprises, or $E_{t-1}p_t = p_t$, neither supply shocks:

$$\log \tilde{Y}_t^A = \frac{a_L}{1-a_L}(p_t - p_t) + a\frac{a_L}{1-a_L} = a\frac{a_L}{1-a_L}$$

By applying Taylor's expansion for the log function and defining the output gap of each sector as $y_t = \dfrac{Y_t - \tilde{Y}_t}{\tilde{Y}_t} \cong \log Y_t - \log \tilde{Y}_t$, the output gap in sector A will be given by:

$$y_t^A = \frac{a_L}{1-a_L}(p_t - E_{t-1}p_t) = \frac{a_L}{1-a_L}[p_t - p_{t-1} - (E_{t-1}p_t - p_{t-1})]$$

In addition, noting that:

$\log P_t - \log P_{t-1} = p_t - p_{t-1} \cong \dfrac{P_t - P_{t-1}}{P_{t-1}} = \pi_t$, and $E_{t-1}(p_t - p_{t-1}) = E_{t-1}p_t - p_{t-1} \cong \dfrac{E_{t-1}P_t - P_{t-1}}{P_{t-1}} = E_{t-1}\left[\dfrac{P_t - P_{t-1}}{P_{t-1}}\right] = E_{t-1}\pi_t$

yields the output gap for sector A below, which is the same as the one obtained in Walsh (2000).

$$y_t^A = \frac{a_L}{1-a_L}(\pi_t - E_{t-1}\pi_t)$$

Appendix C. The Game

The new central banker takes office at period *t* and remains in office until *t*+1. After this period, another central banker is chosen. At each period, wage setters that are renegotiating their contracts will observe the current stance of monetary policy and will update their expectations accordingly.

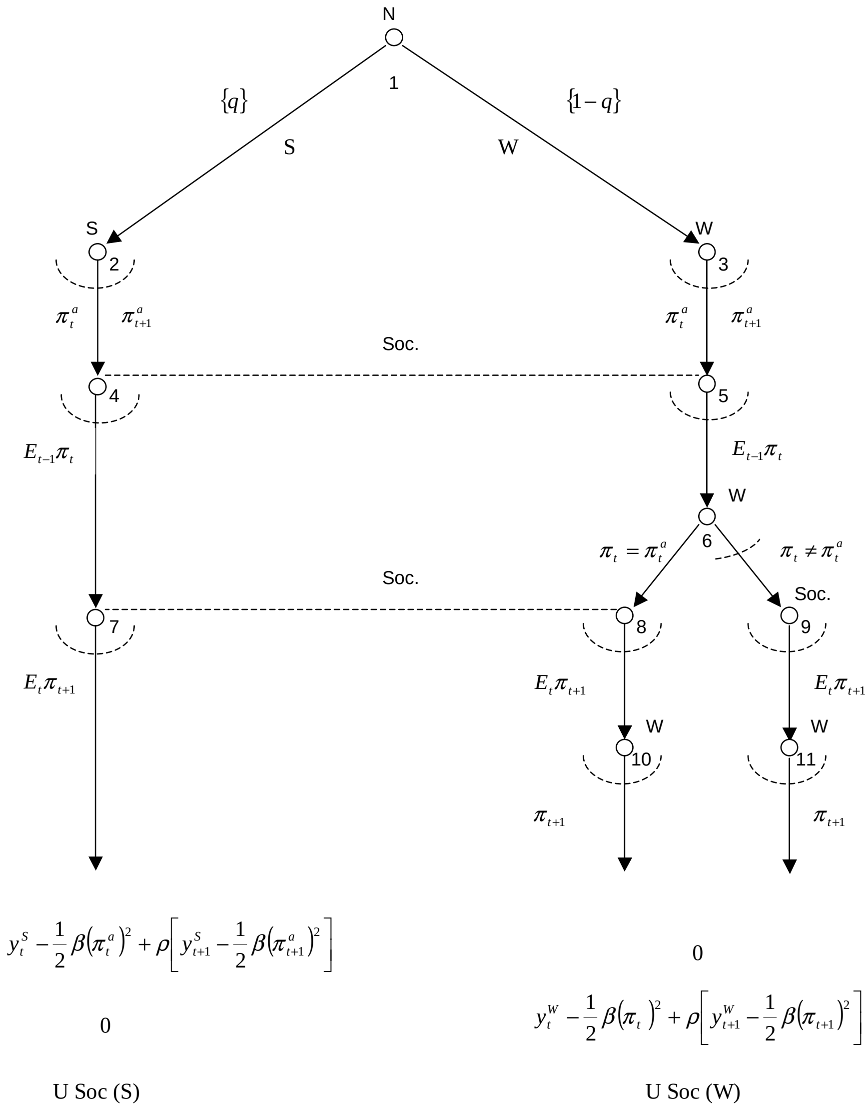
Central banker's utility function is the same for both types and is given by:[13]

$$U^i = \left[y_t - \frac{1}{2}\beta\pi_t^2\right] + \rho\left[y_{t+1} - \frac{1}{2}\beta\pi_{t+1}^2\right]$$

where y_j is the output gap at *j*=*t*, *t*+1, π_j is the actual inflation at *j*=*t*, *t*+1, β is the relative weight attributed to inflation at *t* e *t*+1, and ρ is the intertemporal discount factor ($0 < \rho < 1$).

[13] In spite of the utility function and the relative weights attributed to inflation and output gap being the same for both central bankers, to justify why S fulfills the announced target and W does not, Walsh (2000), on a footnote, proposes that the actual utility function will be given by:
$U^i = [y_t - \beta\pi_t^2] + \rho[y_{t+1} - \beta\pi_{t+1}^2] - \theta^i[\pi_t - \pi_t^a]^2 - \rho\theta^i[\pi_{t+1} - \pi_{t+1}^a]^2$, in which $\theta^W = 0$ and $\theta^S \to \infty$.

Figure 1: The game



Inflation is a direct variable of choice for the central banker. The game begins with nature choosing the central banker. After being chosen, the central banker announces its inflation target for *t* and *t*+1. Cukierman e Liviatan (1991) show that it is strategically optimal for the opportunistic central banker (W) to signal to be of the other type at the beginning for the game, by announcing the same inflation trajectory of a type S central banker. After the announcement, society (Soc) will negotiate the expiring contracts using its most updated expectations regarding the actual inflation trajectory. In this game, society (Soc) is represented by wage setters in the three sectors. The central banker then chooses inflation for period *t* (π_t), which may or may not reveal its type, according to the equilibrium considered. Output and employment then realize. After observing the actual inflation, the wage setters that have contracts expiring at the end of period *t* will update their expectations ($E_t\pi_{t+1}$). The central banker then chooses inflation for period *t*+1 (π_{t+1}). Output and employment at *t*+1 then realize.

Figure 1 shows the game in its extensive form. Dotted lines connecting knots 4 to 5 and 7 to 8 depict information sets, where society cannot distinguish the type of central banker. Dotted semicircles represent infinite possibilities of choice, and only one choice is presented, for didatic purposes. As the conservative central banker always fulfills its announcement, he makes no strategic decisions after the announcement. The original paper presents 3 equilibrium possibilities (pooling, separating, and mixed strategies). The focus of this paper is on the first two. Nonetheless, the occurrence of one or another will be a function of the model's variables. As the game repeats itself at *t+2, t+4,* etc., the stationary equilibrium is what is sought, i.e., the game solution at *t* and *t+1.* Society's utility is represented by *USoc(W)* ou *USoc(S),* in spite of not being modeled in the original paper.

C.1. Separating Equilibrium

In the separating equilibrium, the opportunistic central banker (W) reveals its type at the end of period *t*. To make his choice, he takes agents' expectations as given to maximize his utility. As a result, by backward induction, at *t*+1, type W central banker's problem will be to:

$$\begin{cases} \max_{\pi_{t+1}}\left[y_{t+1}-\frac{1}{2}\beta\pi_{t+1}^{2}\right] \\ \text{s.t. } y_{t+1}=\bar{a}\left[\pi_{t+1}+\frac{1-\gamma}{2}(\pi_t-E_{t-1}\pi_t)-\frac{1+\gamma}{2}E_t\pi_{t+1}-\frac{1-\gamma}{2}\bar{\rho}(E_t\pi_{t+2}-E_{t-1}\pi_{t+1})\right] \end{cases}$$

As all expectations are given, as well as inflation at *t*, the first order condition will yield:

$$\pi_{t+1}^{W}=\frac{\bar{a}}{\beta}=\pi^{d}>0$$

Note that, distinctly from Walsh (2000), $\frac{\partial\pi^d}{\partial\gamma}=0$. That means that the market structure will not affect discretionary inflation at time *t*+1. As a result, at *t*+1, what impacts central banker's decision is solely the linear gain obtained from the increase in inflation impacting the output gap vis-à-vis the (square) cost of an increase in inflation. In Walsh (2000), due to the way expectations are set, γ is part of the parameter that changes the size of the vector of the output gap (α) and therefore positively affects maximum discretionary inflation.

In a separating equilibrium, the opportunistic central banker will choose an inflation rate higher than zero in the second period, in spite of knowing that this inflation rate will be anticipated by those sectors that are renegotiating their contracts at the beginning of time *t*+1. This is the typical argument of these models: subgame perfection makes W choose π^d in the second period. As a result, only those who sign contracts for 2 periods at the beginning of *t* (sector B1) will be surprised. By backward induction, at *t*, the type W central banker will seek to maximize its intertemporal utility, once his choice of π_t will interfere in this utility at time *t*+1 through y_{t+1}. Thus, his problem will be:

$$
\begin{cases}
\max_{\pi_t}\left[y_t-\frac{1}{2}\beta\pi_t^2\right]+\rho\left[y_{t+1}-\frac{1}{2}\beta\pi_{t+1}^2\right] \\
\text{s.t.}\quad
\begin{aligned}
y_{t+1}&=\bar{a}\left[\pi_{t+1}+\frac{1-\gamma}{2}(\pi_t-E_{t-1}\pi_t)-\frac{1+\gamma}{2}E_t\pi_{t+1}-\frac{1-\gamma}{2}\bar{\rho}(E_t\pi_{t+2}-E_{t-1}\pi_{t+1})\right]\\
y_t&=\bar{a}\left[\pi_t+\frac{1-\gamma}{2}(\pi_{t-1}-E_{t-2}\pi_{t-1})-\frac{1+\gamma}{2}E_{t-1}\pi_t-\frac{1-\gamma}{2}\bar{\rho}(E_{t-1}\pi_{t+1}-E_{t-2}\pi_t)\right]
\end{aligned}
\end{cases}
$$

As all expectations and inflation at time t+1 are given, the first order condition will yield:

$$\pi_t^W=-\frac{1}{2}\frac{\alpha}{\beta}(-2-\rho+\gamma\rho)=\left(1+\left(\frac{1-\gamma}{2}\right)\rho\right)\pi^d>\pi^d$$

Note that, distinctly from Walsh (2000), $\pi_t^W\neq\pi_{t+1}^W$ in the separating equilibrium. This result makes sense as the actual unanticipated inflation at *t* affects the output gap not only at time *t*, but also at *t+1*. Therefore, there is an additional incentive to raise inflation at *t*, which was not observed by Walsh (2000) due to the inadequate setting of expectations.

Another result that differs from Walsh (2000) regards the effect of γ upon π_t^W: $\frac{\partial\pi_t^W}{\partial\gamma}=-\frac{\rho}{2}\pi^d<0$ (In Walsh, $\frac{\partial\pi_t^W}{\partial\gamma}>0$). This result suggests that an increase in the share of firms negotiating contracts for one period reduces optimal discretionary inflation at time *t*. A high γ implies that few people will be surprised in the second period, which reduces the incentive described in the previous paragraph. W's choice aims at surprising and he will surprise everyone in the first period, as no one knows for sure who he really is and equilibrium is separating. Therefore, γ does not matter from the viewpoint of surprising society in the first period but it does matter for the degree of surprise achievable in the second period. The effect of the inflation surprise will be higher the more people are stuck in two period contracts signed at the beginning of *t*.

The conservative central banker (S) will take into account the influence of his announcement on the expectations, and thus, will choose the trajectory to be announced by maximizing its intertemporal utility function. His problem will then be:

$$
\begin{cases}
\max_{\pi_t,\pi_{t+1}}\left[y_t-\frac{1}{2}\beta\pi_t^2\right]+\rho\left[y_{t+1}-\frac{1}{2}\beta\pi_{t+1}^2\right] \\
\text{s.t.} \\
y_{t+1}=\bar{a}\left[\pi_{t+1}+\frac{1-\gamma}{2}(\pi_t-E_{t-1}\pi_t)-\frac{1+\gamma}{2}E_t\pi_{t+1}-\frac{1-\gamma}{2}\bar{\rho}(E_t\pi_{t+2}-E_{t-1}\pi_{t+1})\right]\\
y_t=\bar{a}\left[\pi_t+\frac{1-\gamma}{2}(\pi_{t-1}-E_{t-2}\pi_{t-1})-\frac{1+\gamma}{2}E_{t-1}\pi_t-\frac{1-\gamma}{2}\bar{\rho}(E_{t-1}\pi_{t+1}-E_{t-2}\pi_t)\right]\\
\pi_t=\pi_t^a \text{ and } \pi_{t+1}=\pi_{t+1}^a, E_t\pi_{t+1}=\pi_{t+1}^a, E_{t-1}\pi_t=q\pi_t^a+(1-q)\pi_t^W, E_{t-1}\pi_{t+1}=q\pi_{t+1}^a+(1-q)\pi_{t+1}^W
\end{cases}
$$

The corresponding first order condition for π_t^a yields:

$$\pi_t^a = \left[1 + \frac{\rho(1-q)(1-\gamma) - q(1+\gamma)}{2}\right]\pi^d > 0$$

Note that $\pi_t^a < \pi_t^W$, and

$$\frac{\partial \pi_t^a}{\partial \gamma} = -\frac{1}{2}[q + \rho(1-q)]\pi^d < 0$$

The above result is related to the effect of inflation at *t* on the output gap in the second period. In the first period, all agents will be surprised unless $q = 1$. But in the second period only those who signed contracts for 2 periods (t and $t+1$) will be surprised. The higher the share of firms with one period contracts, the less the central banker of type S will have to worry about the recessive effect of low inflation in the separating equilibrium, being thus able to reduce inflation at time *t*. In addition,

$$\frac{\partial \pi_t^a}{\partial q} = -\frac{1}{2}[1 + \gamma + (1-\gamma)\rho]\pi^d < 0$$

The previous result reveals that a higher probability of a type S central banker (higher *q*), allows S to reduce the optimal level of inflation announced. The first order condition for π_{t+1}^a yields:

$$\pi_{t+1}^a = \left(\frac{1}{2(1+\rho)}\right)(1-\gamma)(1-q+\rho(1+q))\pi^d > 0 \text{, if } \gamma \neq 1$$

$$= 0 \text{, if } \gamma = 1$$

which means that, in the general case ($\gamma \neq 1$), inflation will be positive at *t*+1 even under a conservative central banker, contrary to the result obtained in Walsh (2000) where $\pi_{t+1}^a = 0$. Positive inflation at *t*+1 makes sense as there is rigidity at *t*+1 inherited from the decisions taken at the end of *t-1* (or beginning of *t*), which is when sector B1's contracts are signed. At *t*− 1 the agents have not yet observed the type of the central banker and thus negotiate their wage contracts with a positive inflation expectation for *t*+1. Therefore, S will implement some inflation at *t*+1 to accommodate the disadjusments in that sector's expectations. However, $\pi_{t+1}^a < \pi_{t+1}^W$, $\forall q, \rho, \gamma$, and

$$\frac{\partial \pi_{t+1}^a}{\partial \gamma} = \left(\frac{1}{2(1+\rho)}\right)[q(1-\rho) - (1+\rho)]\pi^d < 0$$

which means that with a high γ, there will be contracts being renegotiated after the actual inflation at time *t* is observed, and therefore, there will be no need for higher inflation to inhibit its recessive effect. In addition,

$$\frac{\partial \pi_{t+1}^{a}}{\partial q} = \left(\frac{1}{2(1+\rho)}\right)[\gamma - 1 - \rho(\gamma - 1)]\pi^{d} < 0 \text{, if } \gamma \neq 1$$

$$= 0 \text{, if } \gamma = 1$$

which means that, once agents attribute a higher probability of the central banker being of type S, the optimal inflation for S at *t+1* does not need to be so high. The rationale is similar to $\frac{\partial \pi_{t+1}^{a}}{\partial \gamma}$. Note that $\gamma = 1$ yields the classical result with a one-period model: if the central banker can commit to the announcement, then inflation is null.

Output gap for each type of incumbent will be a function of the optimal choices of inflation in the periods previous and following the current mandate, once expectations transcend central banker's mandate. Under the assumption that the equilibrium is stationary, the separating equilibrium will repeat in the future as it has repeated in the past. For the pooling equilibrium, the same assumption applies. The output gap at *t* in a separating equilibrium will then be:

$$y_t^S = \frac{\pi^d}{1+\rho}\left[\begin{array}{l} \frac{\bar{a}}{2}(2 - q - q\gamma + \rho - \rho q - \rho\gamma + \rho q\gamma)\left(1 - q\rho\left(\frac{1-\gamma}{2}\right) - q\left(\frac{1+\gamma}{2}\right)\right) + \left(\frac{1+\gamma}{2}\right)\bar{a}(1-q)(-2-\rho+\rho\gamma) \\ -\left(\frac{1-\gamma}{2}\right)\rho\left[\begin{array}{l} -\frac{q\bar{a}}{2(1+\rho)}(q - q\gamma - 1 - \rho + \gamma + \rho\gamma - \rho q + \rho q\gamma) \\ +(1-q)\left(\bar{a} + \frac{\bar{a}}{2}(-2-\rho+\rho\gamma)\right) \end{array}\right] \end{array}\right]$$

$$y_t^W = -\pi^d\left(\frac{\bar{a}}{4(1+\rho)^2}\right)\left[\begin{array}{l} \rho^3(2q^2\gamma^2 - 2q^2\gamma + 4\gamma - 2\gamma^2 - 2) + \rho^2(8\gamma - 6q^2\gamma + 2q^2\gamma^2 + q\gamma^2 - q - 5 - 3\gamma^2) \\ + \rho(-3q^2 - 5 + 6\gamma - q + q\gamma^2 - 2q^2\gamma - 3q^2\gamma^2 - \gamma^2) - 2 - q^2\gamma^2 - 2q^2\gamma - q^2 + 2\gamma \end{array}\right] > 0$$

if $q \neq 0$ and $\gamma \neq 1$ jointly.

Note that, even though two hypotheses are used for the calculation of the output gap at *t* (previous central banker of type W or S), the actual type of the former central banker does not affect the output gap in a separating equilibrium. This is due to the fact that at the moment B2 signs contracts, there is no asymmetry of past information: the type of the former central bank will have already been revealed.

The sign of y_t^S will depend on the values of the variables $q, \rho, \gamma, a,$ and β. However, by analyzing a multidimensional n-grid formed by the discrete values that these variables may assume, one concludes that $y_t^S > 0$ whenever $\gamma \leq 0{,}5$, and for $\gamma = 1$ and $q = 0 \text{ or } 1$, $y_t^S = 0$. On the other hand, for $\gamma > 0{,}5$, both negative and positive values were found for y_t^S. Therefore, Walsh (2000)'s findings that a central banker who commits to his targets does not necessarily coexist with economic recessions in the first period of a separating equilibrium are replicated. And as $y_t^W - y_t^S = \bar{a}((1+\gamma) + \rho(1-\gamma))\frac{q}{2}\pi^d > 0$, W achieves higher output expansion from equilibrium levels at period *t* as compared to S. This is possible because $\pi_t^W > \pi_t^S$ and the expectations are an average of the two optimal inflation rates.

At *t*+1:

$$y_{t+1}^S = \pi^d \frac{\rho \bar{a}}{4(1+\rho)^2}(-1+\gamma)\begin{bmatrix} \rho^3\left(-\gamma + q + 2q^2\gamma - 2q^2 + 1 - q\gamma\right) + \rho^2\left(4 + 2q^2\gamma + q\gamma - 6q^2 - q\right) \\ + \rho\left(2 - 3q^2\gamma + 2\gamma + 2q\gamma - 3q^2\right) + 2q - q^2 - q^2\gamma + \gamma - 1 \end{bmatrix}$$

$$y_{t+1}^W = \pi^d \frac{\rho \bar{a}}{4(1+\rho)^2}(-1+\gamma)\begin{bmatrix} \rho^3\left(-\gamma + 2q^2\gamma + 1 - 2q^2\right) + \rho^2\left(q\gamma - 3q - \gamma + 3 - 6q^2 + 2q^2\gamma\right) \\ + \rho\left(q\gamma - 3q^2\gamma - 3q - 3q^2\right) - q^2\gamma - q^2 - 2 \end{bmatrix}$$

in which the sign of y_{t+1}^S and y_{t+1}^W will depend on the combination of the variables $q, \rho, \gamma, a, \text{and } \beta$. Through a numerical analysis of this result, considering a multidimensional n-grid formed by the discrete values of these variables, one concludes that $y_{t+1}^S > 0$ whenever $\gamma \leq 0{,}1$ and $q \geq 0{,}9$, which lies in a very limited and small set of the possible values of q and γ. With $\gamma > 0{,}1$ and $q < 0{,}9$, y_{t+1}^S may be either negative or positive. On the other hand, $y_{t+1}^W > 0$ whenever $\gamma \geq 0{,}4$. In case $\gamma < 0{,}4$, y_{t+1}^W may be either positive or negative. Notice that if $\gamma = 1$, both y_{t+1}^S and y_{t+1}^W will be zero, once there will be no rigidity inherited from the past, leading to expected inflation different from the optimal under the mandate of the revealed central banker. In addition, as $y_{t+1}^W - y_{t+1}^S = \bar{a}\frac{\rho}{1+\rho}(-1+\gamma)\left[\rho^2(q\gamma - q) + \rho(-\gamma - q\gamma - q - 1) - 1 - 2q - \gamma\right]\pi^d > 0$ if $\gamma \neq 1$ $(= 0$ if $\gamma = 1)$, in the more general case output expansion at *t+1* generated by a type W central banker is higher than that achieved by S in a separating equilibrium. This result arises from the fact that a type S central banker takes into account, upon choosing the inflation to be announced, that this announcement affects agents' expectations. He will thus not deviate from the announcement even if he could achieve higher output expansion.

As a result, a smaller share of one period contracts (lower γ) favors output expansion if the central banker is of type S. In contrast, a higher γ favors output expansion for a type W central banker.

W will not deviate from the equilibrium if $U_{separating}^W \geq U_{mimmicS}^W$, and this will occur when:

$$\begin{aligned} &\rho^3\left(q^2 - 2q^2\gamma + q^2\gamma^2\right) + \rho^2\left(-4\gamma - 2q^2\gamma - 2\gamma^2 + 3q^2 - 2 + 2q - 2q\gamma^2 - q^2\gamma^2\right) \\ &+ \rho\left(3q^2 - 2 - 4\gamma - 2\gamma^2 - q^2\gamma^2 + 2q^2\gamma - 2q + 2q\gamma^2\right) + q^2\gamma^2 + q^2 + 2q^2\gamma \geq 0 \end{aligned}$$

Note that the condition for the separating equilibrium to be stable becomes much more complex under the corrected expectations framework.

C.2. Pooling equilibrium

In the pooling equilibrium, a type W central banker will mimic S in the first period. Aware of that, agents will expect inflation at *t* to be equal to the announced. At the end of time *t*, as the agents will not be able to distinguish between the possible types of central bankers, their expectations for inflation at *t+1* will be an average of the maximum discretionary rate and the announced rate, i.e., $E_{t-1}\pi_t = \pi_t^a$ and $E_t\pi_{t+1} = q\pi_{t+1}^a + (1-q)\pi_{t+1}^W$. Thus, type S central banker's problem is:

$$\max_{\pi_t,\pi_{t+1}} \left[y_t - \frac{1}{2}\beta\pi_t^2 \right] + \rho\left[y_{t+1} - \frac{1}{2}\beta\pi_{t+1}^2 \right]$$

s.t.

$$\begin{cases} y_{t+1} = \bar{a}\left[\pi_{t+1} + \frac{1-\gamma}{2}(\pi_t - E_{t-1}\pi_t) - \frac{1+\gamma}{2}E_t\pi_{t+1} - \frac{1-\gamma}{2}\bar{\rho}(E_t\pi_{t+2} - E_{t-1}\pi_{t+1}) \right] \\ y_t = \bar{a}\left[\pi_t + \frac{1-\gamma}{2}(\pi_{t-1} - E_{t-2}\pi_{t-1}) - \frac{1+\gamma}{2}E_{t-1}\pi_t - \frac{1-\gamma}{2}\bar{\rho}(E_{t-1}\pi_{t+1} - E_{t-2}\pi_t) \right] \\ \pi_t = \pi_t^a \text{ and } \pi_t = \pi_t^a, E_{t-1}\pi_{t+1} = E_t\pi_{t+1} = q\pi_{t+1}^a + (1-q)\pi_{t+1}^W, E_{t-1}\pi_t = \pi_t^a \end{cases}$$

The first order condition at *t* yields:

$$\pi_t^a = \left(\frac{1-\gamma}{2} \right)\pi^d > 0, \text{ for } \gamma \neq 1 \qquad \text{(A)}$$
$$= 0, \text{ for } \gamma = 1$$

Note that the announced inflation at *t* in the pooling equilibrium will be lower than that chosen by a type W central banker in a separating equilibrium. In addition:

$$\frac{\partial \pi_t^a}{\partial \gamma} = -\frac{1}{2}\pi^d < 0$$

Indeed, if $\gamma = 1$, the effect of inflation at *t* upon y_t would be null. In this case, everything would work at *t* as in a one-period model with commitment. Therefore, the previous inequality may be interpreted from the viewpoint of B-type contracts. If their share in the total increases, then π_t^a increases, as a result of their effect in the output at period *t* as at *t+1*. The existence of two period contracts is what causes the inertial term to exist at *t*+1. Therefore, an inflationary shock affects y positively. As a result, even a type S central banker benefits from a higher rigidity in contracts.

At *t*+1, the first order condition yields:

$$\pi_{t+1}^a = \frac{1}{1+\rho}(1 - q + \rho(1-\gamma q))\pi^d > 0 \qquad \text{(B)}$$

with $\pi_{t+1}^a < \pi_{t+1}^W$ and $\frac{\partial \pi_{t+1}^a}{\partial \gamma} = -\frac{\rho q}{1+\rho} < 0$. As the equilibrium is pooling, actual inflation at *t*+1 will be lower than the expected average, once agents do not know with certainty the type of the central banker.

Actual inflation at t+1 rises with two period contracts as the expectations of wage setters in sector B1 weight differently from the other sectors (they are multiplied by $\frac{\rho}{1+\rho}$). This is due to the fact that these contracts were signed at the end of t–1 to be in force at t and t+1. If only B2 and A existed, $\frac{\partial \pi_{t+1}^{a}}{\partial \gamma} = 0$, what would make sense as these contracts were renegotiated at the end of t and would be surprised in the same direction.

Equations (A) and (B) show that a conservative central banker will have to inflate at a positive rate (in the more general case) in both periods, being penalized by the uncertainty prevailing at the economy in a pooling equilibrium. Yet, optimal inflation announced for the two periods and implemented by the conservative central banker will be lower than the optimal discretionary rates. In addition, $\frac{\partial \pi_{t+1}^{a}}{\partial \rho} = -\frac{1}{1+\rho}\left[(1-q\gamma)+\frac{1}{1+\rho}(1-q+\rho(1+\gamma q))\right]\pi^{d} < 0$, what suggests that in a situation where the central banker attributes high importance to future outcomes (high ρ), he will tend to reduce inflation in the second period by much more, due to the cost that inflation brings in terms of utility, once it will force agents' expectations downwards as well. Remember that agents' expectations are an average of the announced inflation and the maximum discretionary rate. In addition, $\frac{\partial \pi_{t+1}^{a}}{\partial q} = -\left(\frac{1+\rho\gamma}{1+\rho}\right)\pi^{d} < 0$, i.e., if the probability of a central banker to be conservative increases, society's expectations for inflation at *t+1* will be lower. As a result, the recessive result of lower inflation is reduced.

In the pooling equilibrium, W will inflate at t at the announced rate. At t+1, he will choose inflation strategically. Taking expectations as given, the optimal inflation at t+1 will be equal to that in the separating equilibrium ($\pi_{t+1}^{W} = \pi^{d}$). A deviation from the equilibrium means that W will inflate at t at the rate that maximizes his intertemporal utility, i.e., $\pi_{t}^{W} = \left(1+\rho\left(\frac{1-\gamma}{2}\right)\right)\pi^{d} > \pi^{d}$. By doing that he will reveal his type at the end of the first period. As a result, everybody will be surprised in the first period, as $E_{t-1}\pi_{t} = \pi_{t}^{a}$, but those who can update their expectations for *t+1* will do so according to the newly revealed identity of the central banker, which means that $E_{t}\pi_{t+1} = \pi_{t+1}^{W} = \pi^{d}$. W will choose not to deviate from the pooling equilibrium if and only if $U_{pooling}^{W} \geq U_{deviate}^{W}$. This will occur when:

$$\begin{aligned}&\rho^{3}\left(1-2\gamma+\gamma^{2}\right)+\rho^{2}\left(-2\gamma-4\gamma q^{2}+3-4q^{2}\gamma^{2}-\gamma^{2}\right)\\&+\rho\left(-4\gamma q^{2}+3-\gamma-4q^{2}+2\gamma\right)+1+\gamma^{2}+2\gamma \leq 0\end{aligned}$$

Notice, once more, that the condition for stability of the equilibrium becomes much more complex when the expectational setting is adjusted.

Output gap at t, considering that the pooling equilibrium will repeat in the future and will have repeated in the past (stationary equilibrium), will depend on the type of the central banker, and will be given by

$$y_t^S = y_t^W = -\pi^d\left(\frac{\bar{a}}{4(1+\rho)^2}\right)(-1+\gamma)\begin{bmatrix}\rho^2\left(-2q\gamma+4q^2\gamma-2\gamma\right)+\rho\left(1-2q-2q\gamma+4q^2+2q^2\gamma-3\gamma\right)\\+1-2q+2q^2-\gamma\end{bmatrix},$$ if the former central banker was of a type S, or

$$y_t^S = y_t^W = -\pi^d\left(\frac{\bar{a}}{4(1+\rho)^2}\right)(-1+\gamma)\left[\rho^2\left(4q^2\gamma-2\gamma\right)+\rho\left(1+4q^2+2q^2\gamma-3\gamma\right)+1+2q^2-\gamma\right],$$ if the former central banker was of a W-type.

The sign of y_t in both cases depend on the actual realizations of $q, \rho, \gamma, a,$ and β. However, a numerical analysis performed with an n-grid formed by the discrete values of these variables grants that if $\gamma \neq 1$, $y_t^{\text{previous BC of an S-type}} > 0$ whenever $\gamma < 0{,}3$ and $y_t^{\text{previous BC of type W}} > 0$ whenever $q > 0{,}5$ or $\gamma < 0{,}5$. The output gap at *t* may be either positive or negative for the remaining possible values of γ and *q*.

As $y_t^{\text{previous BC of a W-type}} - y_t^{\text{previous BC of an S-type}} = \bar{a}\left(\frac{1+\rho\gamma}{1+\rho}\right)q(1-\gamma)\pi^d > 0$ for $\gamma \neq 1$, a former central banker of a W-type causes the output gap at *t* to increase in a pooling equilibrium. This is due to the fact that non-anticipated inflation persistence affects output gap at *t*, a result obtained after the informational framework was properly adjusted.

At *t*+1*:*

$$y_{t+1}^S = \frac{\pi^d\bar{a}}{(1+\rho)}\left[\frac{-q+1+\rho-\rho q\gamma}{1+\rho}\left[1-q\left(\frac{1+\gamma}{2}+\rho\left(\frac{1-\gamma}{2}\right)\right)\right]+(1-q)\left[\left(\frac{\gamma-1}{2}\right)\rho+1\right]-\rho\left(\frac{1-\gamma}{2}\right)^2\right] > 0, \forall q,\gamma,\rho,a,\beta$$

$$y_{t+1}^W = \pi^d\,\frac{\bar{a}}{4}\,\frac{\rho}{(1+\rho)^2}\left[\rho^2\left(3-2\gamma+4q^2\gamma^2-\gamma^2\right)+\rho\left(2q^2\gamma^2+5-\gamma^2+6\gamma q^2-4\gamma\right)+2+2\gamma q^2+2q^2-2\gamma\right] > 0,$$ if $q\neq 0$, $\gamma\neq 1$.

The result for y_{t+1}^S differs substantially from the one found in Walsh (2000), in which a central banker committed to his targets always generates recession in the second period of a pooling equilibrium. In addition, as $y_{t+1}^S - y_{t+1}^W = -\pi^d\bar{a}\frac{\rho}{2(1+\rho)}\left[\rho(-3\gamma+q-1+5\gamma q)-\gamma+\gamma q+5q-3\right]$ may be either positive or negative, depending on *q*, an opportunistic central banker will achieve higher output expansion compared to the conservative one if the probability attributed to his being conservative is high enough. That means that he would need to surprise a very large share of society.

The ECO/UnB Working Paper Series

The Department of Economics of the University of Brasilia publishes its Working Papers Series since April 1972. On August 30, 2002 the series was renewed with the on-line publication of the papers. All Working Papers may be freely downloaded from the Department site: http://www.unb.br/ih/eco.

Working papers published since August 2002:

231 Posse de escravos e estrutura da riqueza no agreste e sertão de Pernambuco: 1777-1887. Flávio Rabelo Versiani and José Raimundo O. Vergolino, 30 August 2002, 29p.

232 On the natural rates of unemployment and interest: the Robertson connection. Mauro Boianovsky and John R. Presley, 30 August 2002, 34p.

233 Contas Nacionais e o meio ambiente: reflexões em torno de uma abordagem para o Brasil. Charles C. Mueller, 30 August 2002, 25p.

234 Economics of air pollution: hedonic price model and smell consequences of sewage treatment plants in urban areas. Sérgio A. Batalhone, Jorge M. Nogueira and Bernardo P. M. Mueller, 30 August 2002, 25p.

235 The Brazilian depression of the 80s and 90s. Mirta Bugarin, Roberto de G. Ellery Jr., Victor Gomes and Arilton Teixeira, 30 August 2002, 30p.

236 Informal employment in Brazil – A choice at the top and segmentation at the bottom: a quantile regression approach. Maria Tannuri-Pianto and Donald M. Pianto, 30 August 2002, 23p.

237 False contagion and false convergence clubs in stochastic growth theory. Stephen de Castro and Flávio Gonçalves, 30 August 2002, 20p.

238 Spot and contract markets in the Brazilian wholesale energy market. Paulo C. Coutinho and André Rossi de Oliveira, 30 August 2002, 19p.

239 Tributação da renda e do consumo no Brasil: uma abordagem macroeconômica. Valter Borges de Araújo Neto e Maria da C. S. de Sousa, 30 August 2002, 31p.

240 Vote splitting, reelection and electoral control: towards a unified model. Maurício S. Bugarin. 30 August 2002, 26p.

241 Shadow-prices in payment systems. Rodrigo Peñaloza, 6 September 2002, 31p.

242 Welfare implications of the Brazilian social security system. Roberto de G. Ellery Jr. and Mirta N. S. Bugarin, 13 September 2002, 28p.

243 Os agentes econômicos em processo de integração regional – Inferências para avaliar os efeitos da ALCA. Renato Baumann and Francisco Galrão Carneiro, 13 September 2002, 29p.

244 Leading by example: a simple evolutionary approach. André Rossi de Oliveira and João R. O. de Faria, 20 September 2002, 25p.

245 The role of institutions in sustainable development. Bernardo Mueller and Charles Mueller, 20 September 2002, 25p.

246 Incentivos em consórcios intermunicipais de saúde: uma abordagem de teoria dos contratos. Luciana Teixeira, Maria Cristina MacDowell and Mauricio Bugarin, 27 September 2002, 19p.

247 Liquidity constraints and the behavior of aggregate consumption over the Brazilian business cycle. Mirta Bugarin and Roberto de G. Ellery Jr, 27 September 2002, 19p.

248 Pricing water and sewage services in urban areas: Evidences of low level equilibrium in a developing economy. Ricardo Coelho de Faria, Jorge M. Nogueira and Bernardo Mueller, 4 October 2002.

249 Wrong incentives for growth in the transition from modern slavery to labor markets: Babylon before, Babylon after. Stephen de Castro, 4 October 2002, 23p.

250 Vintage capital, distortions and development. Samuel Pessoa and Rafael Rob, 11 October 2002, 40p.

251 Consórcios intermunicipais de saúde: uma análise à luz da teoria dos jogos. Luciana Teixeira, Maria Cristina MacDowell and Mauricio Bugarin, 11 October 2002, 30p.

252 Preços de escravos em Pernambuco no século XIX. Flávio R. Versiani and José Raimundo O. Vergolino, 18 October 2002, 20p.

253 A model of capital accumulation and rent seeking. Paulo Barelli and Samuel de Abreu Pessoa, 18 October 2002, 47p.

254 Anchors away: the cost and benefits of Brazil's devaluation. Edmund Amann and Werner Baer, 25 October 2002, 20p.

255 Um seguro agrícola "eficiente". Aércio S. Cunha, 25 October 2002, 57p.

256 Campaign contributions with swing voters. Manfred Dix and Rudy Santore, 1 November 2002, 15p.

257 Incentivos para os administradores de empresas estatais: O papel dos dividendos mínimos obrigatórios e o desenho ótimo de salários. André Luís G. Carcia and Maurício Bugarin, 1 November 2002, 28p.

258 Impostos e a História. Aércio S. Cunha, 8 November 2002, 12p.

259 Determinantes do endividamento dos estados brasileiros: Uma análise de dados de painel. Isabela Fonte Boa Rosa Silva e Maria da Conceição Sampaio de Sousa, 8 November 2002, 27p.

260 Technology adoption: On the nonequivalence of tariffs and quotas. Arilton Teixeira, 15 November 2002, 25p.

261 Constitutional regimes, growth and stagnation in the Brazilian economy: 1947-1999. Marco Antônio Campos Martins, 15 November 2002, 39p.

262 Price caps and electoral cycles. César Mattos, 22 November 2002, 16p.

263 Os pobres que levantem a mão (mas será que são mesmo pobres?) - Uma tentativa de validar o cadastro único. Carlos Alberto Ramos and Ricardo Santana, 29 November 2002, 100p.

264 Relative earnings of immigrants and natives under changes in the US wage structure, 1970-1990: A quantile regression approach. Maria Tannuri-Pianto, 29 November 2002, 40p.

265 Bidding strategies in the Brazilian Treasury auctions. Anderson Caputo Silva, 6 December 2002, 34p.

266 Crises cambiais e ataques especulativos no Brasil. Mauro Costa Miranda, 13 December 2002, 26p.

267 Poverty and environment degradation: the Kuznets environmental curve for the Brazilian case. Fabio G. e Barros, Augusto F. Mendonça and Jorge M. Nogueira, 20 December 2002, 27p.

268 On shadow-prices of banks in real-time gross settlement systems. Rodrigo Peñaloza, 20 December 2002, 31 p.

269 A characterization of renegotiation-proof contracts via random fixed points in Banach spaces. Rodrigo Peñaloza, 20 December 2002, 9 p.

270 Existence of time-invariant settlements in FEDWIRE-like payment systems. Rodrigo Peñaloza, 27 December 2002, 13p.

271 Principal-Agent problem with continuum of constraints: the infinite dimensional approach.

Rodrigo Peñaloza, 27 December 2002, 43p.

272 Structural analysis of multiple-unit auctions: recovering bidders' valuations in auctions with dominant bidders. Anderson Caputo Silva, January 3, 2003, 18 p.

273 Financiamento público de campanhas eleitorais: efeitos sobre bem-estar social e representação partidária no Legislativo. Adriana C. Portugal and Maurício S. Bugarin, January 10, 2003, 25p.

274 Wicksell on technical change, real wages and employment. Mauro Boianovsky and Harald Hagemann, January 17, 2003, 28p.

275 Quão pobres são os pobres. Brasil: 1992-2001. Carlos Alberto Ramos and Ricardo Santana, January 24, 2003, 20p.

276 Dois anos da Lei de Responsabilidade Fiscal do Brasil: uma avaliação dos resultados à luz do modelo do fundo comum. Selene Peres Peres Nunes and Ricardo da Costa Nunes, January 31, 2003, 45p.

277 Políticas de geração de emprego e renda: Justificativas teóricas, contexto histórico e experiência brasileira. Carlos Alberto Ramos, February 7, 2003, 28p.

278 Long run implications of the Brazilian capital stock and income estimates. Victor Gomes, Mirta N. S. Bugarin and Roberto Ellery Jr., February 14, 2003, 25p.

279 Taxation on intergenerational bequest and redistribution of wealth in a class-setting. Mauro Baranzini, Sheila Oliveira Benjuino and Joanílio Rodolpho Teixeira, February 21, 2003, 23p.

280 On portfolio management. Paulo Coutinho and Benjamin Miranda Tabak, February 28, 2003, 21p.

281 Decentralized portfolio management. Paulo Coutinho and Benjamin Miranda Tabak, March 7, 2003, 22p.

282 The IS-LM model and the liquidity trap concept: From Hicks to Krugman. Mauro Boianovsky, March 14, 2003, 43p.

283 À espera da reforma orçamentária: um mecanismo temporário para redução de gastos públicos. Carla G. Protásio, Maurício S. Bugarin and Mirta N. S. Bugarin, March 21, 2003, 31p.

284 A dívida pública interna e sua trajetória recente. Flávio Rabelo Versiani, March 28, 2003, 16p.

285 Investment and capital accumulation in Brazil from 1970 to 2000: a neoclassical view. Roberto Ellery Jr., Mirta N. S. Bugarin, Victor Gomes and Arilton Teixeira, April 4, 2003, 20p.

286 Setting the right expectations: a note on Carl Walsh's market discipline paper. Fábia Carvalho and Maurício Bugarin, April 11, 2003, 33p.

Forthcoming working papers:
(Subject to change)

287 The fifth consumer's surplus: An extension of the concept of Marshallian surplus to preferences with non-null income effects. Cassia Helena Marchon and André Luís Rossi de Oliveira, April 18, 2003, 34p.

288 Volume, composição e sustentabilidade da dívida pública de liquidez brasileira no período 1994-2002. Fernando de Aquino Fonseca Neto and Joanílio Rodolpho Teixeira, April 25, 2003, 35p.

289 A tarifa de acesso na indústria de telecomunicação quando a hipótese de monopólio natural é quebrada. Rafael de Melo Silveira and André Rossi de Oliveira, May 2, 2003, 20p.