Universidade de Brasília
Departamento de Economia

Série Textos para Discussão

Accounting for the Hidden Economy: Barriers to Legality and Legal Failures

António R. Antunes
Universidade NOVA de Lisboa

Tiago V. Cavalcanti
Universidade NOVA de Lisboa

Texto nº 299
Brasília, julho de 2003

Department of Economics Working Paper 299
University of Brasilia, July 2003

UNIVERSIDADE DE BRASÍLIA
DEPARTAMENTO DE ECONOMIA

TEXTO PARA DISCUSSÃO Nº 299

Accounting for the Hidden Economy: Barriers to Legality and Legal Failures

António R. Antunes
Universidade NOVA de Lisboa

Tiago V. Cavalcanti
Universidade NOVA de Lisboa

Brasília, 11 de julho de 2003

UNIVERSIDADE DE BRASÍLIA
DEPARTAMENTO DE ECONOMIA
Campus Universitário Darcy Ribeiro
Instituto Central de Ciências
Caixa Postal 04302, 70910-900 Brasília, DF, Brasil
Tel.: (55-61) 3072498, 2723548
Fax: (55-61) 3402311
E-mail: econ@unb.br
http://www.unb.br/ih/eco

Accounting for the Hidden Economy: Barriers to Legality and Legal Failures ⋆

António R. Antunes [a,b], Tiago V. Cavalcanti [a]

[a] *Universidade NOVA de Lisboa, Faculdade de Economia, Lisboa, Portugal*

[b] *Banco de Portugal, Departamento de Estudos Económicos, Lisboa, Portugal*

Abstract

This paper studies a fundamental question in economic development: Why does the size of the informal (unregulated) sector vary so much across countries? In order to address this question it constructs and solves numerically a general equilibrium model with credit constrained heterogenous agents, occupational choices over formal and informal businesses, contractual imperfections (legal failures) and a government sector which imposes taxes and regulations (barriers to legality) on formal firms. The numerical exercises suggest that credit market policies and regulation costs can account for the huge differences in the size of the informal sector across countries, but just part of the differences in output per capita.

JEL Classification: E6; O11; O17

Keywords: Inequality; Credit constraints; Corruption; Informal sector

1 Introduction

This paper studies a fundamental question in economic development: Why does the size of the informal (unregulated) sector vary so much across coun-

⋆ This paper has benefitted from the financial support of Égide at Universidade Nova. We thank Stephen L. Parente, Mário Páscoa, João Maurício Rosal, and Anne Villamil for useful conversation and suggestions. We are also indebted to seminar participants at the Universidade NOVA de Lisboa informal workshop, Universidad de Alicante, Banco de Portugal, and VII Workshop on Dynamic Macroeconomics (Vigo) for helpful discussions and comments. We are responsible for any remaining errors.

Email addresses: ara@fe.unl.pt (António R. Antunes), cavalcan@fe.unl.pt (Tiago V. Cavalcanti).

tries? De Soto (1989) emphasizes that informal production [1] in developing countries is mainly driven by the tremendous difficulties, under the form of regulation and corruption, that entrepreneurs have to acquire legal status. In a recent study (De Soto, 2000), the same author suggests that these *barriers to legality* are not only important to explain differences in the size of the informal sector but also differences in per capita income across countries. The idea is that, without legal status, entrepreneurs cannot exercise full property rights over their assets, and, as a consequence, cannot use their wealth as a collateral to a loan and cannot generate capital from their savings. Due to these *legal failures*, they, in general, under-invest and are locked in to operate low productive technologies.

Our model clearly shares some features of a previous literature [2] on the organization of production and on the process of economic development. Agents in our framework can choose to be either a worker or an (informal or formal) entrepreneur. In this respect, this paper is related to Lucas' (1978) "span of control" model, which was later extended by Rauch (1991) in his study of informal production. Unlike these models, ours is built upon a dynamic framework and uses credit constraints in the analysis of occupational choice. Agents are differentiated by their entrepreneurial ability and their initial wealth. They care about their own consumption and the initial wealth of their offspring. In order to open a formal/informal business, agents must buy in advance the capital to finance their project. However, capital markets are imperfect and not necessarily the best project will be undertaken. This interaction between wealth distribution and capital market imperfection is based on Banerjee and Newman (1993), which was also used by several authors, among whom Lloyd-Ellis and Bernhardt (2000) and Quintin (2001). The former article studies the macroeconomic and distributional dynamics associated with the process of economic development, while the latter investigates how limited enforcement affects the size distribution of firms and productivity across countries. Finally, Dessy and Pallage (2003) also study why the informal sector varies so much across countries. In their model entrepreneurs become legal because they can use productive public infrastructure, while in our model the premium from formalization is better access to outside finance.

Our quantitative experiments suggest that differences in regulation and credit market institutions can account for the huge differences in the size of the informal sector, but just a small part of the difference in output per capita across countries. An economy in which corruption and regulation are comparable to

[1] We adopt Schneider and Enste's (2000) view and define informal activities (the term used by them is shadow economy and in the literature it is also known as underground economy) as those that contribute to the gross national product, but are currently unregistered.

[2] For a comprehensive overview see Schneider and Enste (2000).

the Peruvian economy will be around 2/3 as rich as the United States. Therefore, De Soto (1989) was right, but his subsequent theory explains just part of the differences in international income.

This paper is divided as follows: the next section describes the model economy. Section 3 describes the agents' optimal behavior, defines the competitive equilibrium allocations, and presents some analytical results. Section 4 solves the model numerically and conducts policy experiments. The last section provides some concluding remarks.

2 The model

2.1 Preferences, endowments and technology

2.1.1 Preferences

In each time period ($t = 0, 1, 2, ...$), the economy consists of a continuum of individuals in the unit interval. Each agent lives and is productive for one period, then reproduces another individual so that the population is constant. Agents care about their own consumption and leave bequest to their offspring. Let c_t^i and b_{t+1}^i denote consumption and bequest, respectively, by agent i at period t. Preferences are represented by

$$U^i = (c_t^i)^\gamma (b_{t+1}^i)^{1-\gamma}, \quad \gamma \in (0, 1). \tag{1}$$

This utility function implies that agents are risk-neutral with respect to income as the indirect utility function is linear in wealth. This implies that any additive punishment or reward in utility may be measured in terms of income. Notice that, for tractability, we assume that preferences are for the bequest and not the offspring's utility (see Banerjee and Newman (1993) and Lloyd-Ellis and Bernhardt (2000) for a similar formulation).

2.1.2 Endowments

Each individual can be either a worker or an entrepreneur. Entrepreneurs create jobs and manage their labor force, n. As in Lucas' (1978) "span of control," each individual is endowed with a talent for managing, x^i, drawn from a continuous cumulative probability distribution function $\Gamma(x)$ with finite support $[\underline{x}, \overline{x}]$, where $\underline{x} \geq 0$. Therefore, in each period agents are distinguished by their initial wealth and ability as entrepreneurs, (b_t^i, x_t^i). We assume that the agent's talent for managing is not hereditary. (For notational convenience,

we shall now, and for the remainder of this paper, drop superscript i to denote the agent.)

2.1.3 Production technologies

The technology that managers operate uses labor, n, and capital, k to produce a single consumption good, y, and is represented by

$$y = xk^{\alpha}n^{\beta}, \quad \alpha,\ \beta\ > 0, \quad \text{and} \quad \alpha + \beta < 1. \tag{2}$$

Capital fully depreciates during one period. Managers can operate only one project. Entrepreneurs can choose to declare their establishments (formal sector) or to work in the shadow economy (informal sector). In order to operate in the formal sector, entrepreneurs must pay a start up cost, ς, in the form of regulations and corruption. This cost is assumed to be independent of the firm output since it is an *ex-ante* payment to the government. This is a barrier to become formal. De Soto (1989, 2000) has shown that this cost varies across countries and is especially high in developing countries. Firms that are legally declared also pay an uniform payroll tax, τ. Informal firms do not pay any start up costs and taxes, but pay fines in case they are detected by the tax authority. We assume that the expected punishment rate in the informal sector is a fraction of output, ηy, where $\eta \in [0, 1]$. [3] This is consistent with observers (see Loayza (1996) and De Soto (2000)), who point out that large firms are more easily detected, and informal firms scale down their size to avoid detection.

2.2 The capital market

Agents can borrow capital from a financial intermediary with access to perfect outside capital markets, in which a risk-free bond earns a gross return of $r \geq 1$. Let l be the amount of funds that an agent borrows from the financial intermediary. In order to finance their projects, constrained agents must put up their initial wealth, b, as collateral. Borrowers cannot commit *ex-ante* to their individual promises and can avoid the repayment obligation, rl, by defaulting on their debt and loosing rb. Those that renege on their debt looses the collateral and incur in a cost proportional to what was produced, ϕy. This is equivalent to an additive punishment in utility. This cost reflects the degree that contracts are enforced in the economy. Higher ϕ means a better quality of the legal system. The point here is that, in contrast to Banerjee and Newman

[3] ηy can also be seen as the costs associated to hiding information for operating in the shadow economy. Larger firms incur larger costs in hiding information from the tax authority.

(1993), the quality of the project will be an important determinant of external debt. [4] Since contracts are easily monitored in the formal sector, we assume for simplicity that ϕ is zero in the informal sector. This is consistent with De Soto (2000), who points out that projects and assets in the informal sector are not adequately documented and therefore *"cannot be turned into capital or cannot be used as a collateral for a loan."* Loans will be limited by the agents inheritance and the degree that contracts are enforced.

Notice that resource allocation involves the division of individuals among formal and informal managers and workers, and then the allocation of factors of production among managers. Occupational choices will be driven by the agent's type, (b, x), the efficiency of the capital market, ϕ, and government tax and regulation, τ, η and ς.

3 Optimal behavior and equilibrium

3.1 Entrepreneurs

Those who have enough resources and managerial ability to become entrepreneurs choose the level of capital and the number of employees to maximize profits subject the technological constraint. Since capital markets are imperfect, let us describe the problem of an entrepreneur for a given level of capital k. Let $\mathcal{I}_F$ ($\mathcal{I}_I$) be an indicator function, which takes value one when $j = F$ ($j = I$), and zero otherwise. The problem of an entrepreneur with capital k is

$$\pi_j(k, x; w) = \max_{n_j} (1 - \eta\mathcal{I}_I) x k^\alpha n_j^\beta - (1 + \tau\mathcal{I}_F) w n_j, \tag{3}$$

where $j \in \{F, I\}$. Equation (3) gives the labor demand of each entrepreneur in both sectors:

$$n_j(k, x; w) = \left(\frac{\beta(1 - \eta\mathcal{I}_I) x k^\alpha}{w(1 + \tau\mathcal{I}_F)} \right)^{\frac{1}{1-\beta}}. \tag{4}$$

Substituting (4) into (3) yields the entrepreneur's profit function for a given level of capital,

$$\pi_j(k, x; w) = [(1 - \beta)(1 - \eta\mathcal{I}_I) x k^\alpha]^{\frac{1}{1-\beta}} \left(\frac{\beta}{w(1 + \tau\mathcal{I}_F)} \right)^{\frac{\beta}{1-\beta}}. \tag{5}$$

[4] Several studies have shown (see, in particular, Cohn and Coleman (2000)) that profitability of the firm is an important predictor of external debt, suggesting that lenders may use individual and business characteristics to evaluate projects.

In an environment with perfect enforcement, $\phi = 1$, managers will solve the following problem

$$\max_{k_j \geq 0} \pi_j(k_j, x; w) - r(k_j + \varsigma \mathcal{I}_F). \tag{6}$$

This gives the optimal physical capital level:

$$k_j^*(x; w) = \left[\left(\frac{\beta}{w(1+\tau \mathcal{I}_F)} \right)^{\beta} \left(\frac{\alpha}{r} \right)^{1-\beta} (1 - \eta \mathcal{I}_I) x \right]^{\frac{1}{1-\alpha-\beta}}. \tag{7}$$

Since agents cannot commit to their promises, debt contracts must be self-enforcing. Let a be the amount of capital that is self-financed (or used as a collateral) and l be the amount of funds that is financed in the outside capital market. The income from running a project is

$$V_j(b, x; w) = \max_{0 \leq a_j \leq b,\ l_j \geq 0} \pi_j(a_j + l_j, x; w) - r(a_j + l_j + \varsigma \mathcal{I}_{j=F}) \tag{8}$$

subject to

$$\pi_j(a_j + l_j, x; w) - r(a_j + l_j + \varsigma \mathcal{I}_{j=F}) \geq (1 - \phi \mathcal{I}_{j=F}) \pi_j(a_j + l_j, x; w) - r a_j.$$

This problem yields optimal policy functions $a_j(b, x; w)$ and $l_j(b, x; w)$, and we define the optimal policy function for capital as $k_j(b, x; w) = a_j(b, x; w) + l_j(b, x; w)$. The last restriction is an incentive compatible constraint, which guarantees that individual promises will be fulfilled (see Kehoe and Levine (1993)). We can rewrite this constraint such that

$$l_j(b, x) \leq \left(\frac{\phi}{r} \pi_j(a_j(b, x; w) + l_j(b, x; w), x; w) - \varsigma \right) \mathcal{I}_{j=F}.$$

It can be shown that constrained entrepreneurs put their entire wealth in the project as long as $b \leq k_j^*(x; w)$.[5] This implies that the size of a project of an entrepreneur (b, x) is

$$k_j(b, x; w) \leq b + (\frac{\phi}{r} \pi_F(b + l(b, x; w), x; w) - \varsigma) \mathcal{I}_{j=F}. \tag{9}$$

Therefore, projects are limited by the agents inheritance and the incompleteness of the capital market.

The following lemma summarizes the value of undertaking each project:

[5] See Appendix A.

Lemma 1 *For any $x \in [\underline{x}, \overline{x}]$, and $w > 0$, the value function $V_j(b, x; w)$, and the associated policy function $l_j(b, x; w)$ for $j \in \{F, I\}$ have the following properties:*

1. *$V_j(b, x; w)$ is continuous and differentiable in x and w. If $x > 0$, it is also strictly increasing in x and strictly decreasing in w.*
2. *For $b < k_j^*(x; w)$, $V_j(b, x; w)$ is continuous, differentiable and strictly increasing in b. For $b > k_j^*(x; w)$, $V_j(b, x; w)$ is constant in b. Moreover, $V_j(b, x; w)$ is continuous in $b = k_j^*(x; w)$.*
3. *For all b and x, $l_I(b, x; w) = 0$. $l_F(b, x; w)$ is strictly increasing for $b < k_F^*(x; w)$ and $l_F(b, x; w) = 0$ for $b > k_F^*(x; w)$.*

Proof. See Appendix B. ■

It is important to highlight the *trade-offs* to operate in each sector. In the informal sector entrepreneurs do not pay the payroll tax and the start up cost, but pay a cost in hiding information from the tax authority, and projects are limited by the agents' initial wealth. In the formal sector, managers have access to the financial market, but have to pay tax and costs associated with regulation and corruption.

3.2 Occupational choice

The occupational choice of each agent defines his lifetime income. For any $w > 0$, an agent (b, x) will become an entrepreneur if $(b, x) \in E(w)$, where

$$E(w) = \{(b, x) \in [0, \infty) \times [\underline{x}, \overline{x}] : \max\{V_F(b, x; w), V_I(b, x; w)\} \geq w\}. \quad (10)$$

Let $E^c(w)$ denote the complement set of $E(w)$. Obviously, if $(b, x) \in E^c(w)$, then agents are workers. Among those who are able to operate a business, they will become a formal entrepreneur if $(b, x) \in E_F(w) \subseteq E(w)$, where

$$E_F(w) = \{(b, x) \in E(w) : V_F(b, x; w) \geq V_I(b, x; w)\}. \quad (11)$$

They operate in the informal sector if $(b, x) \in E_I(w) \subseteq E(w)$, where

$$E_I(w) = \{(b, x) \in E(w) : V_I(b, x; w) \geq V_F(b, x; w)\}. \quad (12)$$

The following lemma characterizes the occupational choice for a given bequest and entrepreneurial ability.

Lemma 2 *Define $b_e(x;w)$ as the curve in the (b,x) plane such that $(b,x) \in [0,\infty) \times [\underline{x}, \overline{x}]$ and $\max\{V_F(b,x;w), V_I(b,x;w)\} = w$. Then there exists $x^*(w)$ such that $\frac{\partial b_e(x;w)}{\partial x} < 0$ for $x > x^*(w)$ and $\frac{\partial b_e(x;w)}{\partial x} = -\infty$ for $x = x^*(w)$.*

1. *For all x, if $b < b_e(x;w)$, then $(b,x) \in E^c(w)$.*
2. *For all x, if $b \geq b_e(x;w)$, then $(b,x) \in E(w)$.*

In addition, define $b_s(x;w)$ as the curve in the (b,x) plane such that $(b,x) \in [0,\infty) \times [\underline{x}, \overline{x}]$ and $V_F(b,x;w) = V_I(b,x;w)$.

3. *For all x, if $b \geq b_e(x;w)$ and $b > b_s(x;w)$, then $(b,x) \in E_I(w)$.*
4. *For all x, if $b \geq b_e(x;w)$ and $b \leq b_s(x;w)$, then $(b,x) \in E_F(w)$.*

Proof. See Appendix C. ■

Figure 1 illustrates this lemma. It shows the occupational choice in the (b,x) space for the baseline economy (see parameters on section 4). Lemma 2 and figure 1 suggest that agents are workers when the quality of their project is low, i.e., $x < x^*(w)$ (the lightest shaded area). For $x \geq x^*(w)$, then agents might become entrepreneurs depending if they are credit constrained or not (notice that for very low bequest agents are workers even though their entrepreneurial ability is higher than $x^*(w)$). The negative association between $b_e(x;w)$ and x suggests that managers with better projects need a lower level of initial wealth to run a project. This is rather intuitive since profits are increasing in the quality of the project. Given the low operational costs in the informal sector, unconstrained entrepreneurs will stay illegal. Constrained entrepreneurs will operate in the informal sector, only if the premium from formalization (access to outside finance) is not high enough. Since this premium increases with the quality of the project, only high-productivity projects will operate in the formal sector (darkest shaded area). The area in between the darkest and lightest shaded areas corresponds to managers in the informal sector.

The size of the informal sector depends on the institutional and policy parameters τ, η, ς and ϕ, as well as on distribution Γ. Two limiting cases identify the role of each parameter:

Proposition 3 *For each $w > 0$,*

1. ***(Full Formalization)** if $\eta \geq 1 - \frac{w^{1-\alpha-\beta}}{(1+\tau)^{\beta}(w+r\varsigma)^{1-\alpha-\beta}}$, then $E_I(w) = \emptyset$;*
2. ***(Full Informalization)** if $\eta \leq 1 - \frac{w^{1-\alpha-\beta}}{(1+\tau)^{\beta}(w+r\varsigma)^{1-\alpha-\beta}}$ and $\phi = 0$, then $E_F(b,x) = \emptyset$.*

Proof. The proof follows directly from (8). For each case, just compare the net income $V_j(b,x;w)$ in each sector. ■

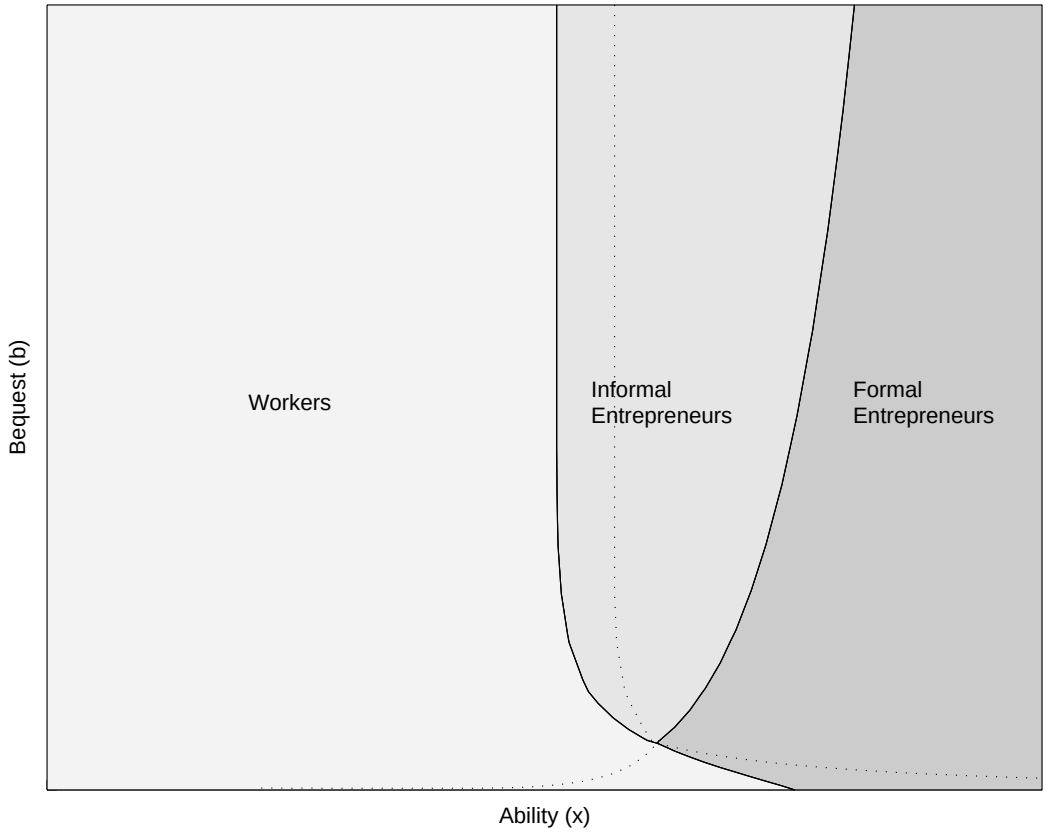

Fig. 1. Firm size distribution in the formal and informal sectors.

Item 1 implies that if the costs of hiding information from the tax authority is too high and corruption is low, then agents have no incentive to undeclare their activities, and there will be no informal activity. On the other hand, item 2 suggests that if the costs associated with the informal sector are low and debt contracts are not enforced, agents will not operate in the formal sector. If $\eta \leq 1 - \frac{w^{1-\alpha-\beta}}{(1+\tau)^{\beta}(w+r\varsigma)^{1-\alpha-\beta}}$, but ϕ is positive, then unconstrained entrepreneurs will prefer to operate in the informal sector. However, formal entrepreneurs might appear in equilibrium, since there is the possibility of outside finance.

3.3 Consumers

At period t, the lifetime wealth of an agent characterized by (b_t, x_t) is given by

$$Y_t = Y(b_t, x_t; w_t) = \max\{w_t, V_F(b_t, x_t; w_t), V_I(b_t, x_t; w_t)\} + rb_t. \tag{13}$$

Lifetime wealth is thus a function of agent-specific b_t and x_t, and economy-wide w_t. Given lifetime wealth, (13), agents choose consumption and bequest to maximize preferences (1). This problem defines the optimal consumption, $c_t = c(Y_t)$, and bequest, $b_{t+1} = b(Y_t)$, policies. The functional form of (1)

implies that agents leave a proportion $1-\gamma$ of their lifetime wealth as a bequest. Notice that bequests cannot be negative because every agent is allowed to become a worker. Define $z_t = (b_t, x_t)$ and let W_t be the bequest distribution at period t.[6]

3.4 Competitive equilibrium

Definition 1 *Given $(\tau, \phi, \eta, \varsigma)$, Γ and W_t, equilibrium at date t is a list w_t, $\{n_j(x; w_t)\}_{j\in\{F,I\}}$, $\{l_j(b, x; w_t)\}_{j\in\{F,I\}}$, $\{a_j(b, x; w_t)\}_{j\in\{F,I\}}$, $\{V_j(b, x; w_t)\}_{j\in\{F,I\}}$, $c_t = c(\cdot)$, $b_{t+1} = b(\cdot)$, such that:*

A. Given the wage rate and government policies, an agent of type (b, x) chooses his occupation to maximize his lifetime wealth, (13).
B. $l_j(b, x; w_t)$ and $a_j(b, x; w_t)$ solve (8) for $j \in \{F, I\}$.
C. Given the lifetime wealth, (13), each agent maximizes utility, (1).
D. Given the wage rate, technology constraint, credit markets, and government policies, formal and informal entrepreneurs select their labor force to maximize profits, (3).
E. The Labor Market clears:

$$\iint\limits_{z\in E_F(w_t)} n_F(x; w_t)W_t(db_t)\Gamma(dx_t) + \iint\limits_{z\in E_I(w_t)} n_I(x; w_t)W_t(db_t)\Gamma(dx_t) = \iint\limits_{z\in E^c(w_t)} W_t(db_t)\Gamma(dx_t). \quad (14)$$

In the quantitative exercises it is important to evaluate policy experiments in "stable" economies, where, for instance, the real wage and income distribution are not changing significantly over time. Indeed, it is possible to show that when policies and institutions are stationary a unique steady-state equilibrium exists (i.e., an equilibrium with a constant real wage, w, and invariant distribution, $H = W\Gamma$) and from any initial condition the economy converges to this equilibrium.

Proposition 4 *There exists a unique stationary equilibrium with $0 < w < \infty$ and invariant distribution W. In addition, for any initial bequest distribution W_0 and stationary government policies and institutions λ, the bequest distribution converges to W.*

Proof. See Appendix D. ■

In the calibration and quantitative experiments we will study the economy in this particular equilibrium and therefore we will consider the long run impact

[6] See the definition of W_t in appendix D.

of changes in policies and institutions.

4 Quantitative results

Before we provide some quantitative results it is important to know the share of informal production relative to total output. Table 1 shows the size of the informal economy relative to GNP for a selected set of countries. [7] Notice that informal production is not only substantial for the set of developing countries, but is also significative for some industrial countries, such as Greece, Italy, Portugal and Spain .

Country	Informal sector size, %
Germany	13.2
US	10
Spain	22.4
Italy	26
Portugal	22.1
Greece	29
Canada	14.8
Brazil[a]	25–35
Peru[a]	40–60
Nigeria[a]	68–76

[a] Average over 1990–93.

Table 1
Informal sector relative to GNP for selected countries in 1995. Source: Schneider and Enste (2000, Tables 2, 3 and 7).

4.1 Parameterization

In order to solve out the model numerically we have to choose a functional form for the ability distribution and assign values to the parameters of the model. We parameterized the model such that, in the stationary equilibrium, we could match some key empirical observations of the United States economy.

[7] See Johnson, Kaufmann, and Zoido-Lobaton (1998), Schneider and Enste (2000), and Friedman, Johnson, Kaufman, and Zoido-Lobaton (2000) for additional data, and for an extensive discussion of the underground economy.

γ	0.8	τ	0.33
β	0.55	ς	0
α	0.35	ϕ	0.25
r	2	η	0.05

Table 2
Parameter values, baseline economy.

Table 2 summarizes the parameter values, which were determined as follows. We interpret the model period to be 35 years and we let $r = 2$, which implies a yearly real interest rate of roughly 2%. We set α and β such that about 55% of income is paid to labor, 35% is paid to the remuneration of capital, and 10% are profits (see Quintin (2001)). We chose a payroll tax of $\tau = 0.33$, which is consistent to the literature (Jones, Manuelli, and Rossi (1993)). Since regulation costs are small in the United States, [8] we assumed that they are negligible relative to lifetime profits of entrepreneurs and we set $\varsigma = 0$. The share of bequest in the instantaneous utility function, $1 - \gamma$, was taken to be 0.2, which is consistent to those estimated by Laitner and Juster (1996). The fraction of output that an entrepreneur can keep in case of default was set to $1 - \phi = 0.75$, which is similar to the one used by Cagetti and De Nardi (2002). Finally, we assumed that the entrepreneurial cumulative distribution function is $\Gamma(x) = Ax^{\frac{1}{\epsilon}}$ and we normalized the support of this distribution to the $[0, 1]$ interval, so that $A = 1$. We chose parameters ϵ and η such that the size of the informal sector and the number of entrepreneurs in the steady-state equilibrium was 10%. [9]

	US economy	Baseline economy
Informal sector size	10	10
Income Gini	40–44	34
% of entrepreneurs	9	9

Table 3
Basic statistics, US and baseline economy. Sources: Schneider and Enste (2000), World Bank (2000) and Cagetti and De Nardi (2002); all figures in percentage.

The baseline economy reproduces statistics consistent to those of the US economy, except for the income Gini coefficient. Since every worker receives the same wage in the model economy, we think that the model income Gini coefficient should underestimate its real world counterpart. The income Gini coefficient when we consider only entrepreneurs income is higher in the data.

[8] According to Loayza (1996) it takes about three to four hours to register a small factory in the United States at almost no income costs.
[9] We defined the size of the informal sector by the share of total output produced by informal firms.

For instance, data in Quadrini (1999) imply a value around 45%. In this case, inequality is close to the one found in the model, which is 49%.

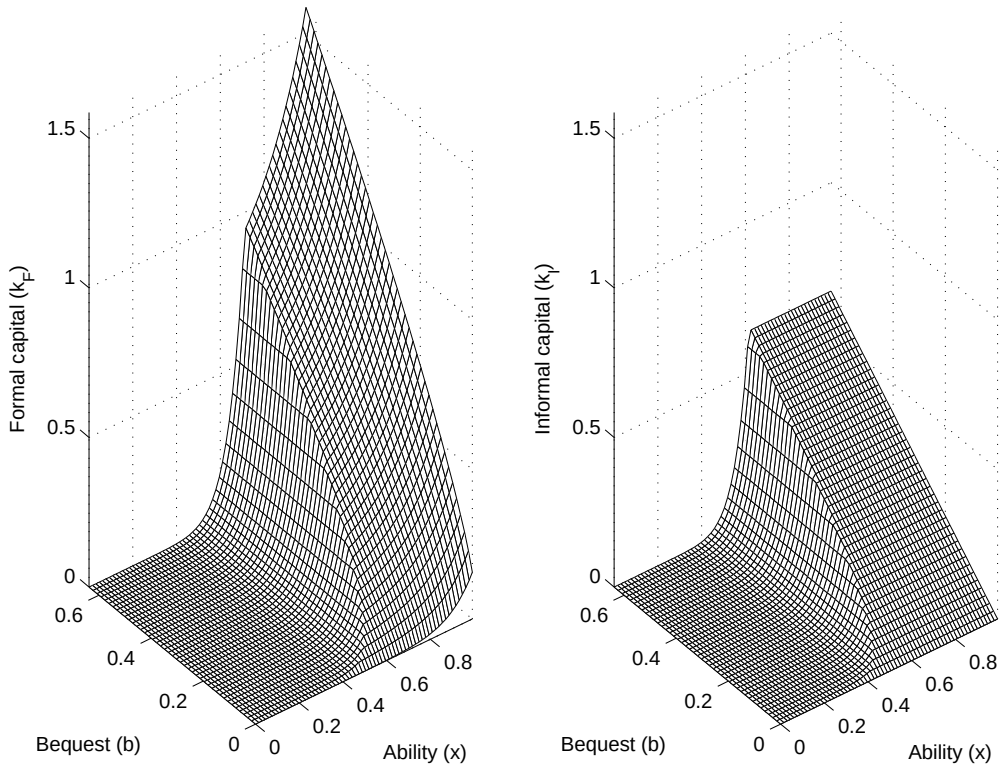

Fig. 2. Firm size in the formal and informal sectors.

Figure 2 shows the distribution of capital allocated in the two sectors. It illustrates well the premium from formalization. The horizontal area (when ability is low) corresponds to those agents that are workers. As the entrepreneurial ability and bequests increase, the size of the projects increases monotonically. The capital in the informal sector, however, is constrained by the initial wealth, which implies that agents with different ability end up having the same level of capital. On the other hand, in the formal sector the total debt increases with the quality of the project. This implies that, for a given level of bequest, the amount of capital employed in this sector is higher for those with better projects. As a consequence, formal entrepreneurs operate more productive technologies.

4.2 Policy experiments

With all parameters determined, we run some policy experiments to investigate the effects of policy changes on, among other things, the size of the informal sector and the allocation of resources. The purpose of these exercises

is to provide a numerical assessment of the long-run impact, for instance, of corruption and credit constraints on the share of production generated outside the realm of government regulation, and on productivity.

	Informal sector size, %	Output per capita	% of entrepreneurs	Income Gini
Baseline	10	100	9	34
$\phi = 0.15$	20	85	11	34
$\phi = 0.35$	6.4	111	7.6	32
$\eta = 0$	22	100	9.6	31
$\eta = 0.08$	5.5	101	8.6	34
$\varsigma = 0.01$	25	95	9.5	33
$\varsigma = 0.05$	64	79	11	33

Table 4
Policy Experiments.

It is important to notice that the effects of these experiments on the size of the informal sector and productivity are not only important qualitatively, but they also deserve quantitative considerations.

The results indicate a quantitatively sizeable impact of financial constraints on the structure of the economy. For instance, when the fraction of output that debtors can keep in case of default increases from 0.75 to 0.85, so that debt contracts are less enforced, the size of the informal sector increases by a factor of 2, and productivity decreases by 15%. When ϕ decreases, then for each x the premium from formalization decreases. This implies that either constrained managers stay undeclared (the informal sector increases) or formal managers scale down their projects (productivity decreases). When $\phi = 0$ there is no premium from formalization and the formal sector is null.

We can also verify that small variations in η can generate substantial differences in the size of the informal sector. The impact of this variable on productivity, however, is small. When η decreases not only the costs to operate in the informal sector decrease (this increases the informal sector which has in general smaller projects), but the productivity in this sector increases. The two effects on productivity roughly offset each other.

Finally, we study the effects of start-up costs in the form of corruption and regulation on the size of the informal sector. As expected, there is a positive relationship between the size of the informal sector and ς. The number $\varsigma = 0.05$

means that this is roughly 31 times the monthly equilibrium wage. [10] De Soto (1989) shows that it took almost a year and 31 times the monthly minimum wage to open a small clothing factory in Peru. Compared to the baseline case, the size of the informal sector in the economy with start up costs comparable to the Peruvian economy is 6 times higher and output per capita is 21% lower. Notice that the size of the informal sector is consistent to the empirical observations for reasonable start up costs (see table 1).

Notice that the model also displays a stylized fact in developing economics: the percentage of entrepreneurs over the total working population decreases with output per capita. Less developed countries tend to have more entrepreneurs but less productive entrepreneurs (Lucas (1978) and Tybout (2000)). When, for instance, enforcement improves, the number of entrepreneurs decreases but the quality (size and productivity) of each project increases.

The model, however, does not generate differences in output per capita consistent to the data. [11] According to the model, a country with regulation costs similar to the Peruvian economy and in which debt contracts are not enforced will be around 2/3 as rich as the United States. [12] Differences in output per capita and total factor productivity (TFP) across countries arises endogenously in the model due to differences in the enforcement system and corruption. The quantitative results suggest that those institutions explain just part of the differences in international incomes.

Parente and Prescott (2000) show that the gap in output per capita among rich and poor countries are explained by differences in total factor productivity (TFP). Their theory is based on technology adoption: the main idea is that inside groups with vested interests block the adoption of more advanced technologies and explain the use of inferior production processes. [13] Our results are complementary to those of Parente and Prescott (2000), since we show that corruption and credit market imperfection can account for part of the differences in TFP across countries. [14]

[10] This is about half of the output per capita.

[11] According to the Penn World table, income per capita among the poorest and richest countries differ by a factor of 30 and Peru in 1990 was 18% as rich as the United States.

[12] For an economy with $\phi = 0.15$, $\eta = 0$, and $\varsigma = 0.05$, the informal sector is roughly 90% and productivity is 73% of the baseline economy. With $\phi = 0$, $\eta = 0$, and $\varsigma = 0.05$, the formal sector is null and output per capita is around 60% of the baseline economy.

[13] This clearly could be extended to explain the presence of institutions that inhibit economy development.

[14] Notice that we could have added in the production process a country specific and exogenous TFP to match the empirical observations on output per capita, but this would not add any new insights to the results.

Finally, we should take the experiments on Table 4 as conservative numbers. The reason is that when, say, corruption increases the informal sector increases, which shrinks the tax base. This in turn could lead to: (i) a rise in taxes to finance spending, which would increase the informal sector, the need of further tax increases, and so on; and (ii) a deterioration of the legal system, since there will be less government resources to spend in any activity. Both (i) and (ii) imply a stronger effect of corruption on the size of the informal sector and productivity.

4.3 Sensitivity analysis

This model does not display wage inequality. This could be introduced by means of a "working ability" that would differentiate among workers. This inequality, however, would simply reflect the randomness of working productivity, which is not important to our purposes. We could have increased inequality by increasing the curvature of the ability distribution (i.e., parameter ϵ). The quantitative exercises are roughly the same for a parameterization with higher inequality (income Gini in range 0.40–0.44). [15]

Table 5 shows some quantitative results when we change other parameters of the model. The second row shows the impact of increasing the lifespan to 45 years. The number of entrepreneurs does not change, whereas, as expected, output increases by 6%.

The third row displays the case where agents are not altruistic. The effect is sizeable on all variables except output. The informal sector is null because there is no bequest in this economy and every entrepreneur need outside resources to undertake it project. Notice, however, that with $\gamma = 1$ and $\phi = 0$ the economy would collapse because everybody would be credit-constrained. In this case, financial constraints could explain any difference in the size of the informal sector and on output across countries. But this is a rather extreme case. The fourth row shows the results for a higher propensity to leave bequest. Output is higher because agents are less credit constrained and as a consequence productivity increases. Notice that the existence of equilibrium requires that $\gamma > 1 - 1/r$. The model therefore displays some sensitivity to parameter γ, but there is no reason to assume that the altruism degree varies across countries.

The last two rows of table 5 consider two extreme cases: one with no enforcement of debt contracts and another with full enforcement. The informal sector size varies from 2.6% to 100%, while output per capita varies from 66% to 124 % of the baseline economy. This confirms our previous findings that

[15] For the sake of space, we omit these results but we can provide them upon request.

credit market policies can account for the differences in the size of the informal sector across countries, but just part of the differences in output per capita. Notice also that financial constraints have a stronger impact on economies with poor enforcement system. [16]

	Informal sector size, %	Output per capita	% of entrepreneurs	Income Gini
Baseline	10	100	9	34
Lifespan = 45	23	106	9.1	29
$\gamma = 1$	0	97	6.2	40
$\gamma = 0.7$	36	106	8.8	27
$\phi = 0$	100	66	12	30
$\phi = 1$	2.6	123	4.5	15

Table 5
Basic statistics, changes in parameters relative to the baseline; all figures in percentage.

5 Concluding remarks

This paper contributes to the literature by characterizing how government policies and institutions interact with the distribution of wealth and entrepreneurial ability in a general equilibrium model with formal and informal sectors, and contractual imperfections. Agents can choose their occupation in two margins: i) become workers or entrepreneurs; ii) operate a formal or an informal business. Formal entrepreneurs pay tax and start up costs in the form of regulation and corruption, while informal entrepreneurs do not pay any regulation and tax, but a cost proportional to output from hiding information from the tax authority. Formal entrepreneurs, however, have a better access to outside finance. The quantitative exercises show that differences in credit market policies and regulation costs can account for the huge differences in the size of the informal sector across countries, but just part of the differences in international incomes.

[16] When ϕ goes from 0 to 0.25, output per capita increases by 40%, while when it goes from 0.25 to 1 output increases by only 24%.

References

Banerjee, A. V., Newman, A. F., 1993. Occupational choice and the process of development. Journal of Political Economy 101 (2), 274–298.

Cagetti, M., De Nardi, M., 2002. Entrepreneurship, frictions and wealth. Federal Reserve Bank of Minneapolis, Working Paper 620 .

Cohn, R., Coleman, S., 2000. Small firms' use of financial leverage: Evidence from the 1993 National Survey of Small Business Finance. Journal of Business and Entrepreneurship 12 (3), 87–103.

De Soto, H., 1989. The Other Path. Harper and Row.

De Soto, H., 2000. The mistery of capital: why capitalism triumphs in the West and fails everywhere? Basic Books, New York.

Dessy, S., Pallage, S., 2003. Taxes, inequality and the size of the informal sector. Journal of Development Economics 70 (1), 225–233.

Friedman, E., Johnson, S., Kaufman, D., Zoido-Lobaton, P., 2000. Dodging the grabbing hand: The determinants of unofficial activity in 69 countries. Journal of Public Economics 76, 459–493.

Hopenhayn, H. A., Prescott, E. C., 1992. Stochastic monotonicity and stationary distributions for dynamic economies. Econometrica 60 (6), 1387–1406.

Johnson, S., Kaufmann, D., Zoido-Lobaton, P., 1998. Regulatory discretion and the unofficial economy. American Economic Review 88, 387–392.

Jones, L. E., Manuelli, R. E., Rossi, P. E., 1993. Optimal taxation in models of endogenous growth. Journal of Political Economy 101, 485–517.

Kehoe, T., Levine, D., 1993. Debt-constrained asset markets. Review of Economic Studies 60, 865–888.

Laitner, J., Juster, T., 1996. New evidences on altruism: A strudy of TIAA-CREF retirees. American Economic Review 86 (4), 893–908.

Lloyd-Ellis, H., Bernhardt, D., 2000. Inequality and economic development. Review of Economic Studies 67 (1), 147–168.

Loayza, N., 1996. The economics of the informal sector: a simple model and some empirical evidence from Latin America. Carnegie-Rochester Conference Series on Public Policy 45, 129–162.

Lucas, Jr, R. E., 1978. On the size distribution of business firms. Bell Journal of Economics 9 (2), 508–523.

Parente, S. L., Prescott, E. C., 2000. Barriers to riches. MIT Press, Cambridge, Massachusetts.

Quadrini, V., 1999. The importance of entrepreneurship for wealth concentration and mobility. The Review of Income and Wealth 45.

Quintin, E., 2001. Limited enforcement and the organization of production. Mimeo, Federal Reserve Bank of Dallas.

Rauch, J. E., 1991. Modelling the informal sector formally. Journal of Development Economics 35 (1), 33–48.

Schneider, F., Enste, D., 2000. Shadow economies: size, cause and consequences. Journal of Economic Literature 38 (1), 77–114.

Stokey, N. L., Lucas, Jr, R. E., 1989. Recursive Methods in Economic Dynam-

ics. Harvard University Press, Cambridge, Massachusetts, with Edward C. Prescott.

Tybout, J., 2000. Manufacturing firms in developing countries: How well do they do, and why? Journal of Economic Literature 28 (1), 11–44.

World Bank, 2000. World Development Indicators. World Bank, Washington D.C.

A Kuhn-Tucker conditions for problem (8)

The Lagrangean associated with problem (8) is

$$\begin{aligned} L_j = {} & \pi_j(a_j + l_j, x; w) - r(a_j + l_j + \varsigma \mathcal{I}_{j=F}) \\ & + \lambda_j[\mathcal{I}_{j=F}\phi\pi_j(a_j + l_j, x; w) - r(l_j + \varsigma\mathcal{I}_{j=F})] + \chi_j[b - a_j]. \end{aligned}$$

The Kuhn-Tucker conditions are:

$$\frac{\partial L_j}{\partial l_j} = \pi_{j1}(a_j + l_j, x; w) - r + \lambda_j[\mathcal{I}_{j=F}\phi\pi_{j1}(a_j + l_j, x; w) - r] \leq 0, \quad \text{(A.1)}$$

$$\frac{\partial L_j}{\partial a_j} = \pi_{j1}(a_j + l_j, x; w) - r + \lambda_j[\mathcal{I}_{j=F}\phi\pi_{j1}(a_j + l_j, x; w)] - \chi_j \leq 0, \quad \text{(A.2)}$$

$$\lambda_j[\mathcal{I}_{j=F}\phi\pi_j(a_j + l_j, x; w) - r(l_j + \mathcal{I}_{j=F}\varsigma)] = 0, \quad \text{(A.3)}$$

$$\chi_j[b - a_j] = 0, \quad \text{(A.4)}$$

$$l_j \geq 0, \quad \frac{\partial L_j}{\partial l_j} l_j = 0, \quad a_j \geq 0, \quad \frac{\partial L_j}{\partial a_j} a_j = 0, \quad \lambda_j \geq 0, \quad \chi_j \geq 0,$$

along with the incentive compatible constraint and the upper limit on a_j. If the entrepreneur is credit constrained, $\lambda_j > 0$, that is, he would be better off if the credit constraint were eased. Notice first that, from (A.3), $l_I = 0$. We know that $\pi_{F1}(k_F^*(x; w), x; w) = r$ and $\pi_{F1}(a_F + l_F, x; w)$ is decreasing with l_F. Notice that $a_F + l_F \leq k_F^*(x; w)$, since $k_F^*(x; w)$ is the unconstrained optimal level of capital. Then, equation (A.2) implies $\chi_F > 0$, which implies by (A.4) that $a_F = b$.

B Proof of Lemma 1

Continuity of $V_j(b, x; w)$ follows from the *Maximum Theorem* and differentiability from Theorem 4.11 of Stokey and Lucas (1989). From the envelope theorem it is easily seen that, provided $x > 0$,

$$V_{j2}(b, x; w) = \pi_{j2}(b + l_j(b, x; w), x; w)(1 + \lambda_j\phi\mathcal{I}_{j=F}) > 0,$$

$$V_{j3}(b,x;w) = \pi_{j3}(b + l_j(b,x;w), x; w)(1 + \lambda_j \phi \mathcal{I}_{j=F}) < 0,$$

If $b \leq k^*(x;w)$, then

$$V_{j1}(b,x;w) = \pi_{j1}(b + l_j(b,x;w), x; w)(1 + \lambda_j \phi \mathcal{I}_{j=F}) > 0.$$

When $b > k_j^*(x;w)$, then by definition of $k_j^*(x;w)$ the net income from entrepreneurship cannot increase and $V_{j1}(b,x;w) = 0$. $l_I(b,x;w) = 0$ since there is no borrowing in the informal sector. For $b > k_F^*(x;w)$ it is also obvious that $l_F(b,x;w) + a_F(b,x;w) = k_F^*(x;w)$. When agents are credit constrained, the incentive compatible constraint holds with equality and

$$\phi \pi_F(b + l_F(b,x;w), x; w) = r(l_F(b,x;w) + \varsigma).$$

Thus,

$$\frac{\partial l_F(b,x;w)}{\partial b} = \frac{\phi \pi_{F1}(k_F,x;w)}{r - \phi \pi_{F1}(k_F,x;w)}.$$

By condition (A.1), we have that $r - \phi \pi_{F1}(k_F,x;w) = \frac{\pi_{F1}(k_F,x;w) - r}{\lambda_F}$. Since this is for constrained agents, $\lambda_F > 0$ and, as we have seen previously, $\pi_{F1}(k_F,x;w)$ is greater than r. Therefore,

$$\frac{\partial l_F(b,x;w)}{\partial b} = \lambda_F \frac{\phi \pi_{F1}(k_F,x;w)}{\pi_{F1}(k_F,x;w) - r} > 0.$$

C Proof of Lemma 2

If agents have sufficiently high b and

$$\max_{j \in \{I,F\}} \{V_j(b,x;w)\} \geq w,$$

there is $x^*(w)$ such that for $x < x^*(w)$ agents prefer to be workers rather than managers:

$$x^*(w) = \min_{j \in \{I,F\}} \left\{ \left(\frac{r}{\alpha}\right)^{\alpha} \left(\frac{w(1 + \tau \mathcal{I}_{j=F})}{\beta}\right)^{\beta} \left(\frac{w + r\varsigma \mathcal{I}_{j=F}}{1 - \alpha - \beta}\right)^{1-\alpha-\beta} \frac{1}{1 - \eta \mathcal{I}_{j=I}} \right\}.$$

$x^*(w)$ is independent of b. For constrained agents with $x \geq x^*(w)$, we have that $\max_{j \in \{I,F\}} \{V_j(b,x;w)\} = w$ defines $b_e(x;w)$, such that

$$\frac{\partial b_e(x;w)}{\partial x} = -\frac{V_{j2}(b,x;w)}{V_{j1}(b,x;w)},$$

where $j = \arg\max_{j \in \{I,F\}} \{V_j(b,x;w)\}$, in all points where $b_e(x;w)$ is differentiable. This is negative from Lemma 1.

Define

$$G(b,x;w) = V_F(b,x;w) - V_I(b,x;w).$$

Provided $G_1(b,x;w) \neq 0$, by the implicit function theorem $G(b,x;w) = 0$ defines $b_s(x;w)$, where

$$\frac{\partial b_s(x;w)}{\partial x} = -\frac{V_{F2}(b,x;w) - V_{I2}(b,x;w)}{V_{F1}(b,x;w) - V_{I1}(b,x;w)}.$$

We have

$$\begin{aligned} V_{F2}(b,x;w) - V_{I2}(b,x;w) = {} & x^{\frac{1}{1-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} \times \\ & \left[\left(\frac{(b+l)^\alpha}{(1+\tau)^\beta}\right)^{\frac{1}{1-\beta}} (1+\lambda_F\phi) - (b^\alpha(1-\eta))^{\frac{1}{1-\beta}}\right] \\ V_{F1}(b,x;w) - V_{I1}(b,x;w) = {} & \alpha x^{\frac{1}{1-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} \times \\ & \left[\left(\frac{(b+l)^\alpha}{(1+\tau)^\beta}\right)^{\frac{1}{1-\beta}} \frac{(1+\lambda_F\phi)}{b+l} - \frac{(b^\alpha(1-\eta))^{\frac{1}{1-\beta}}}{b}\right]. \end{aligned}$$

Notice that $V_F(b,x;w) \geq V_I(b,x;w)$ implies that

$$x^{\frac{1}{1-\beta}} \left(\frac{\beta}{w}\right)^{\frac{\beta}{1-\beta}} \left[\left(\frac{(b+l)^\alpha}{(1+\tau)^\beta}\right)^{\frac{1}{1-\beta}} - (b^\alpha(1-\eta))^{\frac{1}{1-\beta}}\right] \geq \frac{r(l+\varsigma)}{1-\beta} > 0.$$

Since $\lambda_F \geq 0$, this implies that the numerator of $\frac{\partial b_s(x;w)}{\partial x}$ is always positive for $V_F(b,x;w) \geq V_I(b,x;w)$. For a given b, l increases and λ decreases with x, which implies that for sufficiently high x the denominator is negative and $\frac{\partial b_s(x;w)}{\partial x}$ is positive (see Figure 1).

D Proof of Proposition 4

First we need to show that, for every bequest distribution, there exists a finite equilibrium wage rate that clears the labor market. Given the bequest and ability distributions, W and Γ, define the excess demand function $ED(w)$ by

$$\begin{aligned} ED(w) = {} & \iint\limits_{z \in E_F(w)} n_F(x;w) W(db)\Gamma(dx) \\ & + \iint\limits_{z \in E_I(w)} n_I(x;w) W(db)\Gamma(dx) - \iint\limits_{z \in E^c(w)} W(db)\Gamma(dx). \end{aligned} \tag{D.1}$$

The excess demand $ED(w)$ is continuous since both $n_j(x;w)$ and $V_j(b,x;w)$ are continuous in w (see equation (4) and Lemma 1). In addition, $n_j(x;w)$ and $V_j(b,x;w)$ are also strictly decreasing in w. Notice that as w goes to zero, no agent wants to become a worker, $V_j(b,x;w)$ is unbounded and $ED(w) > 0$. Analogously, when w increases, then $ED(w) < 0$. Therefore, by continuity of $ED(w)$ there must be some w^* such that $ED(w^*) = 0$.

It remains to show that $w^* \in [\underline{w}, \overline{w}]$, where $\underline{w} > 0$ and $\overline{w} < \infty$. Let us consider an initial bequest distribution that assigns zero bequest to all agents. Set $E_F(w)$ is the measure of all agents for sufficiently small w as long as $\phi > 0$. In this case, the equilibrium wage rate, $\underline{w}$, is positive and finite, as stated in the previous paragraph. Since the wage rate is positive, next periods' bequests will all be positive. Therefore, the set of possible occupational choices cannot shrink, and might even expand. This implies that for the previous wage rate $\underline{w}$, the excess demand is nonnegative, $ED(w) \geq 0$, which in turn means that for this new bequest distribution the wage rate that clears the labor market is $w' \geq \underline{w}$. Consequently, $\underline{w} > 0$ is the lowest equilibrium wage rate for any initial distribution.

Now suppose an initial bequest distribution that assigns $\overline{b}_0$ to all agents such that $\overline{b}_0 \geq k_j^*(\overline{x};\underline{w})$. By the first argument in this proof, there exists a positive and finite equilibrium wage rate, $\overline{w} < \infty$. In this case, no agent is credit constrained. Either the smallest bequest, $(1-\gamma)(\overline{w} + r\overline{b}_0)$, is higher than $\overline{b}_0$, in which case the next periods' equilibrium wage rate will be the same; or it is smaller than $\overline{b}_0$ and the set of occupation choices might shrink. Therefore, for this new wealth distribution $ED(\overline{w}) \leq 0$. In this case the new equilibrium wage rate is $w' \leq \overline{w}$.

We can thus conclude that $w_t \in [\underline{w}, \overline{w}]$ for all t. The maximum possible bequest is thus $\overline{b}$ such that

$$\overline{b} = (1-\gamma)(\max_{j=F,I}\{\pi_j(k_j^*(\overline{x};\underline{w}),\overline{x};\underline{w})\} + r\overline{b}), \tag{D.2}$$

where we assume that $(1-\gamma)r < 1$. On the other hand the minimum bequest is

$$\underline{b} = (1-\gamma)(\underline{w} + r\underline{b}) \tag{D.3}$$

Define $Z = [\underline{b}, \overline{b}]$ and $z_t = (b_t, x_t)$. Z is compact. Define the measurable space $(Z, \mathcal{B})$, where $\mathcal{B}$ is the Borel algebra for the set. Define $\Lambda(Z, \mathcal{B})$ as the set of all possible probability measures defined on the measurable space $(Z, \mathcal{B})$. For instance, W_t, which specifies the probability of each event in $\mathcal{B}$ at time t, belongs to $\Lambda(Z, \mathcal{B})$. Measure W_t defines a non-stationary transition probability function,

$$P_t(b_t, A) = \Pr\{b_{t+1} \in A | b_t\},$$

for any (b_t, A) in $(Z, \mathcal{B})$. Function P_t assigns a probability to event A for the

descendant of an agent that has bequest b_t but does not know yet x_t. We want to show that the operator $T^* : \Lambda(Z, \mathcal{B}) \to \Lambda(Z, \mathcal{B})$ defined as

$$(T^* W_t)(A) = \int P_t(b_t, A) W_t(db_t), \tag{D.4}$$

where P_t is the transition function defined above, has a unique fixed point $T^*W = W$ for any Borel subset $A \in \mathcal{B}$, given the initial bequest distribution W_0. $(T^*W_t)(A)$ can be interpreted as the probability that the next period's state lies in A according to the present period's distribution. Of course, $T^*W_t = W_{t+1}$. Notice first that w_t is well defined for every distribution W_t, as we argued previously. Second, we know that $b_{t+1} = h(z_t; w_t)$, where $h(z_t; w_t) = (1-\gamma)Y(z_t; w_t)$ (see equation 13), is increasing in z_t for any w_t, and Z is compact. Operator $(Tf)(b_t) = \int f(b_{t+1}) P_t(b_t, db_{t+1})$, defined for any bounded function $f : B(Z) \to B(Z)$, where $B(Z)$ is the set of real-valued bounded functions defined on Z, is the conditional expectation of function f at $t+1$ given that the state at t is b_t. Since, for any wage rate $w_t \in [\underline{w}, \overline{w}]$, $h(z_t, u_t)$ is bounded and increasing in b_t, and x_{t+1} is independent of b_t, the conditional expectation of $f(b_{t+1})$ on b_t is also increasing and bounded provided that f is increasing. Intuitively, this means that, given the equilibrium wage rate w_t, an agent's descendant would never be worse off in terms of the expected value of b_{t+1} if, for any $\varepsilon > 0$, the agent's state were $b_t + \varepsilon$ instead of b_t. As function Tf is increasing, T^* is increasing and P_t is a monotonic transition function.[17] By Corollary 2 of Hopenhayn and Prescott (1992), there is a fixed point for map T^*.

It remains to show that P_t satisfies the Monotone Mixing Condition (MMC). First, define $P_{t+n}(b_t, A) = \Pr\{(b_{t+n}) \in A | b_t\}$. This is the n-step transition function beginning at t. We must show that the transition function P_{t+n} satisfies, for all t,

$$P_{t+N}(\underline{b}, [b_a, \overline{b}]) > \epsilon \quad \text{and} \quad P_{t+N}(\overline{b}, [\underline{b}, b_a]) > \epsilon$$

for some $b_a \in Z$, $\epsilon > 0$, and $N \in \mathbb{N}$. Let us, for simplicity and without loss of generality, omit subscript t. Let w be the wage rate associated with the fixed point of map T^*, W. Define the minimum stationary bequest b_l such that $b_l = (1-\gamma)(w + r b_l)$. Let $b_a = (1-\gamma)(w + r b_l) + \varrho$ for some small $\varrho > 0$. We now show that there is a positive probability that the N^{th} descendent of an agent with $b = \underline{b}$ receives a bequest above b_a. Notice first that the agent's descendants will have bequest in the vicinity of b_l in finite time because they will all be workers. Since the measure of sets $E(w)$ and $E^c(w)$ is non-zero and constant (as the labor market clears with wage in $[\underline{w}, \overline{w}]$), and ability is independent across generations, there is a positive probability that a worker becomes entrepreneur and vice-versa. Suppose that agents with ability in the

[17] See Stokey and Lucas (1989, pages 220 and 379).

vicinity of $\overline{x}$ and bequest in the vicinity of b_l cannot have descendants that become entrepreneurs. Since all agents' descendants face a positive probability of having bequest in the vicinity of b_l in finite time (as they can have successive low x's), this implies that the measure of agents (workers) in the vicinity of b_l is 1, a contradiction to the fact that $E(w)$ has non-zero measure. Therefore, agents with ability in the vicinity of $\overline{x}$ and bequest in the vicinity of b_l have descendants that become entrepreneurs. Moreover, they can become so in the following generation. This implies that they can also have bequest higher than $b_a > b_l$ as long as they have a sufficiently high x, in which case they have high credit limits. Starting from $b = \overline{b}$ is easier: a succession of low x's leaves the agent's descendants with bequest lower than b_a, as they will become workers and remain so until one of them gets a sufficiently high x. Therefore, by Theorem 2 of Hopenhayn and Prescott (1992), there exists a unique time invariant distribution W and associated equilibrium wage w, such that from any initial distribution W_0, the operator T^*W_t converges to W.

The ECO/UnB Working Paper Series

The Department of Economics of the University of Brasilia publishes its Working Papers Series since April 1972. On August 30, 2002 the series was renewed with the on-line publication of the papers. All Working Papers may be freely downloaded from the Department site: http://www.unb.br/ih/eco.

Working papers published since August 2002:

231 Posse de escravos e estrutura da riqueza no agreste e sertão de Pernambuco: 1777-1887. Flávio Rabelo Versiani and José Raimundo O. Vergolino, 30 August 2002, 29p.

232 On the natural rates of unemployment and interest: the Robertson connection. Mauro Boianovsky and John R. Presley, 30 August 2002, 34p.

233 Contas Nacionais e o meio ambiente: reflexões em torno de uma abordagem para o Brasil. Charles C. Mueller, 30 August 2002, 25p.

234 Economics of air pollution: hedonic price model and smell consequences of sewage treatment plants in urban areas. Sérgio A. Batalhone, Jorge M. Nogueira and Bernardo P. M. Mueller, 30 August 2002, 25p.

235 The Brazilian depression of the 80s and 90s. Mirta Bugarin, Roberto de G. Ellery Jr., Victor Gomes and Arilton Teixeira, 30 August 2002, 30p.

236 Informal employment in Brazil – A choice at the top and segmentation at the bottom: a quantile regression approach. Maria Tannuri-Pianto and Donald M. Pianto, 30 August 2002, 23p.

237 False contagion and false convergence clubs in stochastic growth theory. Stephen de Castro and Flávio Gonçalves, 30 August 2002, 20p.

238 Spot and contract markets in the Brazilian wholesale energy market. Paulo C. Coutinho and André Rossi de Oliveira, 30 August 2002, 19p.

239 Tributação da renda e do consumo no Brasil: uma abordagem macroeconômica. Valter Borges de Araújo Neto e Maria da C. S. de Sousa, 30 August 2002, 31p.

240 Vote splitting, reelection and electoral control: towards a unified model. Maurício S. Bugarin. 30 August 2002, 26p.

241 Shadow-prices in payment systems. Rodrigo Peñaloza, 6 September 2002, 31p.

242 Welfare implications of the Brazilian social security system. Roberto de G. Ellery Jr. and Mirta N. S. Bugarin, 13 September 2002, 28p.

243 Os agentes econômicos em processo de integração regional – Inferências para avaliar os efeitos da ALCA. Renato Baumann and Francisco Galrão Carneiro, 13 September 2002, 29p.

244 Leading by example: a simple evolutionary approach. André Rossi de Oliveira and João R. O. de Faria, 20 September 2002, 25p.

245 The role of institutions in sustainable development. Bernardo Mueller and Charles Mueller, 20 September 2002, 25p.

246 Incentivos em consórcios intermunicipais de saúde: uma abordagem de teoria dos contratos. Luciana Teixeira, Maria Cristina MacDowell and Mauricio Bugarin, 27 September 2002, 19p.

247 Liquidity constraints and the behavior of aggregate consumption over the Brazilian business cycle. Mirta Bugarin and Roberto de G. Ellery Jr, 27 September 2002, 19p.

248 Pricing water and sewage services in urban areas: Evidences of low level equilibrium in a developing economy. Ricardo Coelho de Faria, Jorge M. Nogueira and Bernardo Mueller, 4 October 2002.

249 Wrong incentives for growth in the transition from modern slavery to labor markets: Babylon before, Babylon after. Stephen de Castro, 4 October 2002, 23p.

250 Vintage capital, distortions and development. Samuel Pessoa and Rafael Rob, 11 October 2002, 40p.

251 Consórcios intermunicipais de saúde: uma análise à luz da teoria dos jogos. Luciana Teixeira, Maria

Cristina MacDowell and Mauricio Bugarin, 11 October 2002, 30p.

252 Preços de escravos em Pernambuco no século XIX. Flávio R. Versiani and José Raimundo O. Vergolino, 18 October 2002, 20p.

253 A model of capital accumulation and rent seeking. Paulo Barelli and Samuel de Abreu Pessoa, 18 October 2002, 47p.

254 Anchors away: the cost and benefits of Brazil's devaluation. Edmund Amann and Werner Baer, 25 October 2002, 20p.

255 Um seguro agrícola "eficiente". Aércio S. Cunha, 25 October 2002, 57p.

256 Campaign contributions with swing voters. Manfred Dix and Rudy Santore, 1 November 2002, 15p.

257 Incentivos para os administradores de empresas estatais: O papel dos dividendos mínimos obrigatórios e o desenho ótimo de salários. André Luís G. Carcia and Maurício Bugarin, 1 November 2002, 28p.

258 Impostos e a História. Aércio S. Cunha, 8 November 2002, 12p.

259 Determinantes do endividamento dos estados brasileiros: Uma análise de dados de painel. Isabela Fonte Boa Rosa Silva e Maria da Conceição Sampaio de Sousa, 8 November 2002, 27p.

260 Technology adoption: On the nonequivalence of tariffs and quotas. Arilton Teixeira, 15 November 2002, 25p.

261 Constitutional regimes, growth and stagnation in the Brazilian economy: 1947-1999. Marco Antônio Campos Martins, 15 November 2002, 39p.

262 Price caps and electoral cycles. César Mattos, 22 November 2002, 16p.

263 Os pobres que levantem a mão (mas será que são mesmo pobres?) - Uma tentativa de validar o cadastro único. Carlos Alberto Ramos and Ricardo Santana, 29 November 2002, 100p.

264 Relative earnings of immigrants and natives under changes in the US wage structure, 1970-1990: A quantile regression approach. Maria Tannuri-Pianto, 29 November 2002, 40p.

265 Bidding strategies in the Brazilian Treasury auctions. Anderson Caputo Silva, 6 December 2002, 34p.

266 Crises cambiais e ataques especulativos no Brasil. Mauro Costa Miranda, 13 December 2002, 26p.

267 Poverty and environment degradation: the Kuznets environmental curve for the Brazilian case. Fabio G. e Barros, Augusto F. Mendonça and Jorge M. Nogueira, 20 December 2002, 27p.

268 On shadow-prices of banks in real-time gross settlement systems. Rodrigo Peñaloza, 20 December 2002, 31 p.

269 A characterization of renegotiation-proof contracts via random fixed points in Banach spaces. Rodrigo Peñaloza, 20 December 2002, 9 p.

270 Existence of time-invariant settlements in FEDWIRE-like payment systems. Rodrigo Peñaloza, 27 December 2002, 13p.

271 Principal-Agent problem with continuum of constraints: the infinite dimensional approach. Rodrigo Peñaloza, 27 December 2002, 43p.

272 Structural analysis of multiple-unit auctions: recovering bidders' valuations in auctions with dominant bidders. Anderson Caputo Silva, January 3, 2003, 18 p.

273 Financiamento público de campanhas eleitorais: efeitos sobre bem-estar social e representação partidária no Legislativo. Adriana C. Portugal and Maurício S. Bugarin, January 10, 2003, 25p.

274 Wicksell on technical change, real wages and employment. Mauro Boianovsky and Harald Hagemann, January 17, 2003, 28p.

275 Quão pobres são os pobres. Brasil: 1992-2001. Carlos Alberto Ramos and Ricardo Santana, January 24, 2003, 20p.

276 Dois anos da Lei de Responsabilidade Fiscal do Brasil: uma avaliação dos resultados à luz do modelo do

fundo comum. Selene Peres Peres Nunes and Ricardo da Costa Nunes, January 31, 2003, 45p.

277 Políticas de geração de emprego e renda: Justificativas teóricas, contexto histórico e experiência brasileira. Carlos Alberto Ramos, February 7, 2003, 28p.

278 Long run implications of the Brazilian capital stock and income estimates. Victor Gomes, Mirta N. S. Bugarin and Roberto Ellery Jr., February 14, 2003, 25p.

279 Taxation on intergenerational bequest and redistribution of wealth in a class-setting. Mauro Baranzini, Sheila Oliveira Benjuino and Joanílio Rodolpho Teixeira, February 21, 2003, 23p.

280 On portfolio management. Paulo Coutinho and Benjamin Miranda Tabak, February 28, 2003, 21p.

281 Decentralized portfolio management. Paulo Coutinho and Benjamin Miranda Tabak, March 7, 2003, 22p.

282 The IS-LM model and the liquidity trap concept: From Hicks to Krugman. Mauro Boianovsky, March 14, 2003, 43p.

283 À espera da reforma orçamentária: um mecanismo temporário para redução de gastos públicos. Carla G. Protásio, Maurício S. Bugarin and Mirta N. S. Bugarin, March 21, 2003, 31p.

284 A dívida pública interna e sua trajetória recente. Flávio Rabelo Versiani, March 28, 2003, 16p.

285 Investment and capital accumulation in Brazil from 1970 to 2000: a neoclassical view. Roberto Ellery Jr., Mirta N. S. Bugarin, Victor Gomes and Arilton Teixeira, April 4, 2003, 20p.

286 Setting the right expectations: a note on Carl Walsh's market discipline paper. Fábia Carvalho and Maurício Bugarin, April 11, 2003, 33p.

287 The fifth consumer's surplus: An extension of the concept of Marshallian surplus to preferences with non-null income effects. Cassia Helena Marchon and André Luís Rossi de Oliveira, April 18, 2003, 32p.

288 Volume, composição e sustentabilidade da dívida pública de liquidez brasileira no período 1994-2002. Fernando de Aquino Fonseca Neto and Joanílio Rodolpho Teixeira, April 25, 2003, 25p.

289 A tarifa de acesso na indústria de telecomunicação quando a hipótese de monopólio natural é quebrada. Rafael de Melo Silveira and André Rossi de Oliveira, May 2, 2003, 20p.

290 Metas de déficit: transferências intergovernamentais e o controle do endividamento dos estados. Henrique A. Pires and Maurício Bugarin, May 9, 2003, 19p.

291 Jackstrapping DEA scores for robust efficiency measurements. Borko D. Stošić and Maria da C. S. de Sousa, May 16, 2003, 20p.

292 Monopoly rights can reduce income big time. Berthold Herrendorf and Arilton Teixeira. May 23, 2003, 33p.

293 Property rights, violence and the State. Lee J. Alston and Bernardo Mueller. May 30, 2003, 26p.

294 Technical efficiency of the Brazilian municipalities: correcting non-parametric frontier measurements for outliers. Maria da C. S. de Sousa and Borko D. Stošić, June 6, 2003, 23p.

295 Violence in the capital of Brazil: an analysis based on the economic model of crime. André Luís Rossi de Oliveira, June 13, 2003, 15p.

296 Steady state analysis of an open economy general equilibrium model for Brazil. Mirta Noemí Sataka Bugarin, Roberto de Goes Ellery Jr, Victor Gomes Silva, Marcelo Kfoury Muinhos, June 20, 2003, 29p.

297 Benefit sharing: an incentive mechanism for social control of government expenditure. Maurício S. Bugarin, Laércio Mendes Vieira, Leice Maria Garcia, June 27, 2003, 22p.

298 A structural economic dynamic approach to technological gaps. Ricardo A. Araujo and Joanilio R. Teixeira, July 4, 2003, 18p.

299 Accounting for the hidden economy: barriers to legality and legal failures. António R. Antunes and Tiago V. Cavalcanti, July 11, 2003, 24p.